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EFFECTS OF LONGITUDINAL IMPACT ON THE FRACTURE OF ROUND PLASTER RODS

by

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INTRODUCTION

The analytical abilities of many present day engineeers are utilized in studying the response of structures to impulsive loading. For centuries mankind has recognized and has utilized the consequences of the impact of one body on another, the shock of a pressure wave, or the release of a suddenly tapped potential source, but extensive knowledge necessary for design of structural elements subjected to impact is inadequate. The reasons for the lack of such knowledge in this area are numerous. Transient phenomena associated with the fleeting nature of impulsive loading have been difficult to observe and record. The values of many mechanical properties as determined by means of the conventional tests are not applicable to this type of loading, and the values of some of the properties which do apply have not yet been obtained. Also, systems such as those in which equilibrium does not prevail involve more variables. The interrelationship between the many disciplines which are involved in the analysis of the phenomena are only now being recognized.

Technological advancements, however, in the last two decades have revealed the need for research in this area. Technology has also provided the tools for conducting the investigations. The design of structures and materials to resist failure under impulsive loading is one of the more challenging problems in engineering at the present time.

Research of this nature can commence with simple structures, and in this investigation of fracture as a result of impact, a slender, cylindrical rod was chosen as the object of observation. The choice of a slender rod permitted the use of the elementary theory, and this theory was found to be quite satisfactory in the prediction of pulse behavior in the rod. The impulsive loading was created by longitudinal impact which resulted in uniaxial tensile and compressive stresses. In order to study the character of the resulting fracture, the largest diameter of rod which could be suitably handled in the laboratory was selected. The outcome of the selective procedures was a structural specimen which was geometrically a right circular cylinder of 1½-inch diameter and 24-inches in length.

The material of the specimen was a high-grade plaster, commercially marketed under the tradename "Hydrocal B-11". It was used principally for three reasons. First, a material was wanted whose properties would approach those of an ideally brittle material, i. e., one which is elastic and obeys Hooke's law to fracture. In this way plastic behavior in the specimen would not complicate the laboratory investigation or the theoretical analysis. Secondly, most structures built for blast protection of personnel (and therefore subject to impulsive loading) are made of reinforced concrete, and the properties of plaster resemble in many ways those of concrete. Thus, a material was used which would behave in a manner closely approximating one used in the practical application. Thirdly, much research has been done at Iowa State University using plaster as the resistant material. Data on mixing and mechanical properties are available in several recent theses.

A knowledge of the basic theory of pulse transmission and reflection permitted the preliminary selection of the machinery and allied apparatus to be used. The impact producing machine previously used by Dr. C. W. Martin at Iowa State University was modified to accommodate the 1½-inch diameter specimens. Certain mechanical properties such as the velocity of longitudinal pulse propagation and Young's modulus were assumed initially, and the basic components of the machinery were selected on that basis. The assumed properties were corrected after the results of the early tests were known, and more precise investigations were then instituted.

A careful design of equipment was required to insure the occurrence of events in their proper sequence. For instance, after the impact of one steel rod against a second one, the strain pulses which were generated moved along the system consisting of the two steel rods and a plaster specimen. At a section midway along the specimen of plaster two tensile pulses were superimposed. Here the strain intensity was sufficient to cause fracture. The lengths of the rod components of the system were quite critical. If these were of the proper length, the strain would be built up in the plaster rod at the section where it could be most advantageously observed.

Whenever a strain pulse encounters a change in geometry or passes from one medium to another, its intensity is affected. The ratio of the strain in the transmitted pulse to that of the primary pulse may be defined as the transmissibility factor of the device causing the intensity change. It is recognized that in many instances a low transmissibility factor is desirable while in others a large factor may be required. The greater penetration of steel piling compared with wood piling may be explained, for example, on the basis of its lower transmissibility factor. Of principal interest in this investigation was the value of this factor in regard to the interface between the steel and plaster rods. Results show that the theoretical and experimental values of this transmissibility factor were in close agreement.

The strains produced by the impulsive loading were measured by electrical resistance strain gages which were located on the surface of the rod. Consideration was given to the gage length, to the method of securing the gage to the surface, and to its sensitivity. Previous investigations had shown that where the strain pulse has a length equivalent to eight rod diameters strains measured on the rod surface closely compared to the average strain over the cross section. This condition was met in this investigation. The gages were incorporated in a bridge whose output was fed into an oscilloscope. Oscillograms whose coordinate axes were strain versus time were produced by photographing the face of the oscilloscope.

Fracture of the specimen was identified by the use of conductive circuits located on and within the plaster rod. In one set of experiments 4 circuits running longitudinally were employed. On the surface the circuit appeared as an element of the cylinder; a circuit located internally was along the axis of symmetry of the cylinder. By this arrangement it was hoped to locate the origin of the crack and to observe some of the characteristics of its propagation. Fracture of the material was assumed to occur at the immediate vicinity of the circuit at the same instant that the circuit was broken. The circuit was incorporated in a bridge and the output fed into a channel of the oscilloscope. Thus, the time at which fracture occurred was determined for 4 separate points on the cross section.

In the last set of experiments it was desired to ascertain the time at which fracture of the cross section was completed. To accomplish this the entire surface of the rod in the vicinity of the expected break section was coated with the conductive paint. Observations in the case of this one conductive circuit were made in the same manner as had been made previously with the 4 conductive circuits.

Since brittle materials are in general much stronger in compression than in tension, compressive pulses may be transmitted great distances without fracture occurring enroute. Arriving at a free surface, however, this pulse is reflected as a tensile pulse, and if of sufficient intensity, fracture will occur. This was the nature of the loading produced in this investigation. A compressive pulse was produced by the impact, but it was of such low intensity that a compressive failure would not result. Two reflected tensile pulses met at a pre-selected cross section where their superimposed intensities caused the specimen to fracture.

In the earlier investigations the possibility of the fracture having been caused by a single crack which formed at some discontinuity and whose tip expanded in a circular front on the cross section was considered. If such were the case, from information obtained from the 4 fracture circuits, the time interval for the crack to propagate along 3 paths became known. This information would then allow the determination of three unknowns, namely, the crack propagation velocity and the two coordinates of the origin of the crack.

If, however, rather than a single crack originating at a discontinuity and propagating uninterrupted across the section, a number of cracks could originate independently, simultaneously, and join to complete the fracture, then a random breakage of the conductive circuits would occur. Crack propagation velocities could not be obtained if there were multiple breakage on the cross section.

Experimental results confirmed that the conductive circuits did break at random. Several independent cracks did indeed form, were propagated so as to join together, and after a time lapse, the fracture of the section was complete.

In summary, the laboratory investigation is concerned with an impact applied to a system. Consideration is given as to how the resulting impulse is propagated through the system and to why fracture occurs where it does. The ability of the material to sustain a great stress momentarily is explained. The lapse of time between the application of the stress and the resulting fracture is measured. The character of the fracture is perceived.

SURVEY OF LITERATURE

The behavior of a material when subjected to impulsive loading is quite different than when the loading is statically applied. This difference has been noted and has been the stimulus for laboratory research by various investigators for many years. As far as it can be determined from a literature survey, the pioneering investigator in the laboratory as to the effects of impulsive loading was the British scientist, Dr. John Hopkinson (1).

As early as 1872 he investigated the effect of the blow of a weight on the lower end of a long wire. A hole was drilled through the weight, and the wire threaded through the hole. The wire was fixed at the upper end, and a stop was provided at the lower end. The experiment consisted of determining the least distance various weights should fall in order to fracture the wire of 27 ft length. Although the wire had a static tensile strength of 350 lb, any of the weights ranging from 7 to 41 lb would break the wire near the bottom when dropped a distance of 5 ft. All weights except that of 7 lb would break the wire at the top when dropping only 2 ft.

Dr. Hopkinson expressed his surprising results in these words, "In problems of this kind it has been assumed by some that two blows were equivalent when their vis vivas were equal, by others when the momenta were equal; my result is that they are equivalent when the velocities or heights of fall are equal."

In the early part of the 20th century Bertram Hopkinson (2), the son of Dr. John Hopkinson, repeated the experiment that his father had performed. He incorporated some improvements in technique because he sought additional information. He wished to find whether the wire under these conditions of loading could momentarily withstand a stress greater than the static breaking strength, and his results show that the wire could indeed do this and without any great deviation from the elastic behavior.

By far the greatest contribution of the younger Hopkinson, however, was the conception of an ingenious device for measuring forces of short duration. The Hopkinson (3) pressure bar, a name by which the device has since become known, consists of two cylindrical bars of the same diameter and material and which are connected by a joint which will transmit compression but will separate whenever tension is applied. Such a device is shown in Fig. 1.

If a compression pulse is propagated along the pressure bar A, it will travel through the joint and into the shorter bar B without change of shape. Upon reflection as a tension pulse at the right end of B, it again seeks to travel through the joint as seen in the lower drawing of Fig. 1. When the tension pulse reaches the joint, bar B flies off while bar A remains where it is. By catching the bar B in a ballistic pendulum, the momentum of the bar can be measured. The momentum of B equals the area under the force time plot for the pulse.

If the length L of the pulse is greater than twice the length of Bar B, then obviously all of its momentum will not be contained in B. Hopkinson therefore varied the length of B, commencing with very short lengths and increasing in steps until the momentum that he measured became constant. In this manner of proceeding he was able to produce a somewhat rough force-time record for the pulse.

Although the use of this procedure represented a great step in the determination of the characteristics of a pulse propagated along a bar, it suffered from some limitations. The method does not give a complete force-time record of the pulse. Moreover, the dispersive characteristics of the pulse cannot be studied since its character is known only at one location. Hopkinson attempted to study the distortion as a result of the dispersive characteristics, but he obtained no quantitative information.

Landon and Quinney (4) continued the work of the younger Hopkinson after his death. They were particularly interested in the distortion of the pulse after it had traveled along the rod. They attacked the problem by producing the same pulse in each instance but by varying the length of the pressure bar A. The length of the shorter bar B, however, they kept fixed. They observed that the momentum of the short bar decreased with a lengthening of the pressure bar, and thus they concluded that the intensity of the pulse decreased as it propagated along the bar.

During the last 20 years the study of pulse propagation in bars has received a new impetus because of the development of electronic equipment capable of making the precise measurements required. First to utilize the more modern equipment in this work was R. M. Davies (5). Essentially, he replaced the short bar of Hopkinson's apparatus with a condenser type electric strain gage which was located on the same end that the short bar had been located. The signal from the gage was introduced to the deflector plates of an oscilloscope where it was plotted along a time axis. The result was photographed, and an oscillogram of strain versus time was observed for the first time. This allowed the complete picture of the

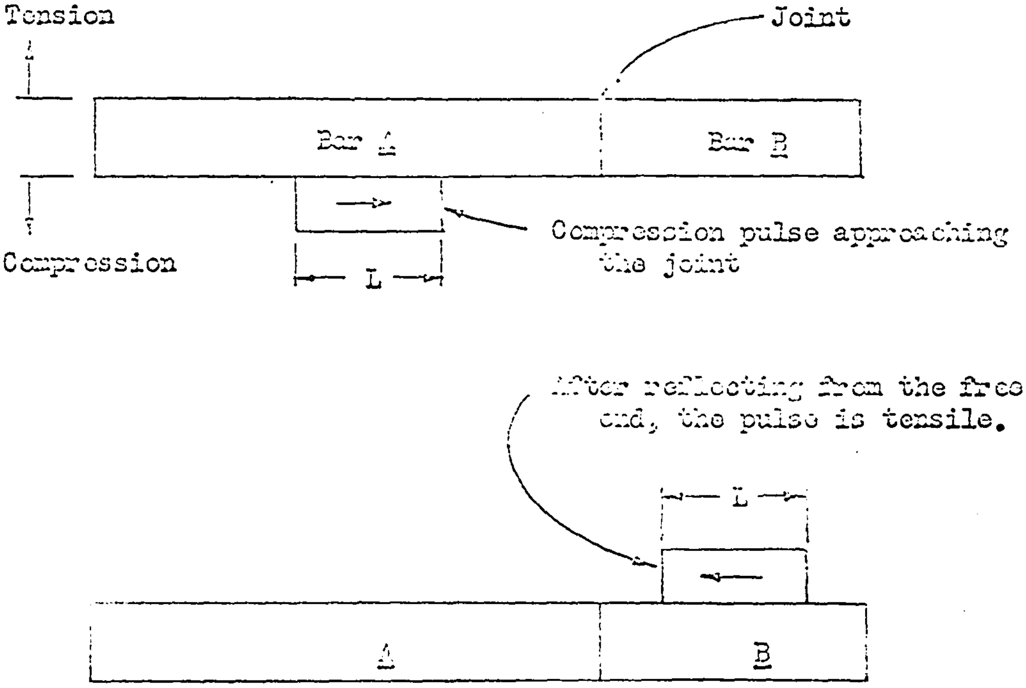


Fig. 1. The Hopkinson pressure bar

pulse form to be scrutinized.

Davies did not place gages at stations along the pressure bar, and he was not able to study the form of the pulse during its journey. Instead, he computed the character of the pulse applied at one end of the bar and compared that to the oscillogram that he had produced at the other end. In this way he revealed the distortion resulting from the travel along the bar.

The location of the fracture resulting from impulsive loading is dependent upon the velocity of pulse propagation in the material. Since a pulse may be represented as a Fourier expansion of sinusoidal waves, the understanding of pulse propagation may commence with the development of the theory of wave transmission in solids. The partial differential equation of wave motion was first treated by D'Alembert in 1750. His interests lay in determining the motion of a vibrating string. In the case of a vibrating bar Lord Rayleigh (6) in his book, "Theory of Sound," devoted a chapter to longitudinal and torsional vibrations. This book was published in 1877, and in it are derived the wave equation for longitudinal displacements and the velocity of propagation of the longitudinal wave.

The elementary theory underlying Lord Rayleigh's derivations was based on the assumptions that plane sections remained plane and that a uniform tensile or compressive stress existed on the cross section of the bar. Radial inertia due to the transverse displacements was neglected. His expression for the velocity of wave propagation was $c = (E/\rho)^{\frac{1}{2}}$, in which $\underline{E}$ is Young's modulus and $\underline{\rho}$ is the density of the material of the bar.

Since the intensity of the wave is affected by a change in mechanical or physical properties of the bar, Lord Rayleigh derived a convenient relationship between the intensity of the reflected wave and that of the incident wave at such a discontinuity. Although he gave no relationship between the intensity of the transmitted wave and that of the incident wave, his method can be readily extended so as to obtain this relationship.

In 1930 Donnell (7) presented a valuable paper on the solution of complex problems on pulse propagation by the use of some simplifying procedures. He pointed out that if the material obeyed Hooke's law and if frictional resistances were negligible, the principle of superposition could be applied in a convenient and workable manner. He established the laws which applied to the simple cases, and he replaced the complex problems by a number of the equivalent simpler ones. Among other applications by which his analysis was demonstrated were the effects of longitudinal pulses which were produced by the braking and throttling of railway trains.

In 1953, Ripperger (8) observed the pulses propagated by striking the ends of slender steel rods with steel ball bearings. Since he wished to measure extremely small strains, he employed piezoelectric strain gages, which have a high sensitivity. The gages were attached to the surface of the rods because he wished to know the suitability of surface strain measurements for studying the propagation of pulses. His work reveals that a pulse can be transmitted along a bar without serious distortion if the length of the pulse as it travels along the rod is equivalent to eight rod diameters. He refrained from answering definitely whether surface strain measurements could be suitably used for studying the short, i. e., less than eight rod diameters, pulses. He found that the pulse traveled at the theoretical velocity even though its duration was as short as 6 microseconds and that the velocity was independent of the rod diameter.

Graham and Ripperger (9) investigated the possibility of determining the average strain on the cross section by the measurement of surface strains. Pulses were produced by impacting steel balls on the end of a slender aluminum rod. The average strains were measured by means of a quartz crystal sandwiched in the cylindrical rod. Surface strain measurements of the pulse were determined by the use of strain gages of the SR-4 type.

Their results reveal that while surface strain measurements may not give a good indication of the average strain in the rod for the short pulses, it can conveniently and accurately be used for the long pulses. Here again the length of the long pulse is equivalent to at least 8 times the rod diameter, or it may be described as one which is propagated with negligible dispersion. These authors recommended the use of the SR-4 gages cemented to the surface for satisfactory measurement of long pulses.

Skalak (10) solved the longitudinal impact problem of two semi-infinite elastic rods of circular cross section, each approaching the other at a velocity, v. The initial conditions of the problem are shown in Fig. 2. His approximate solution has the form of simple integrals of the Airy function, and it shows a dispersion of the pulse front over an

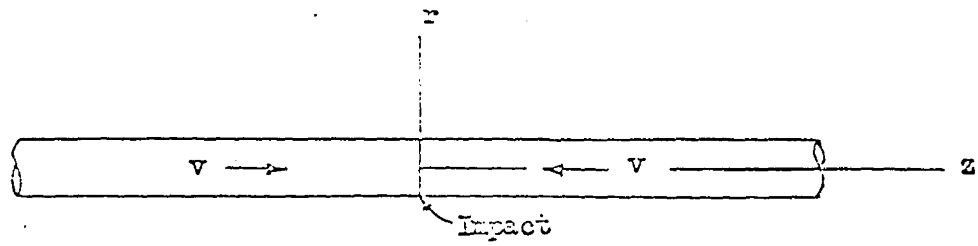


Fig. 2. Initial conditions of the impact of two semi-infinite rods--from Skalak (10)

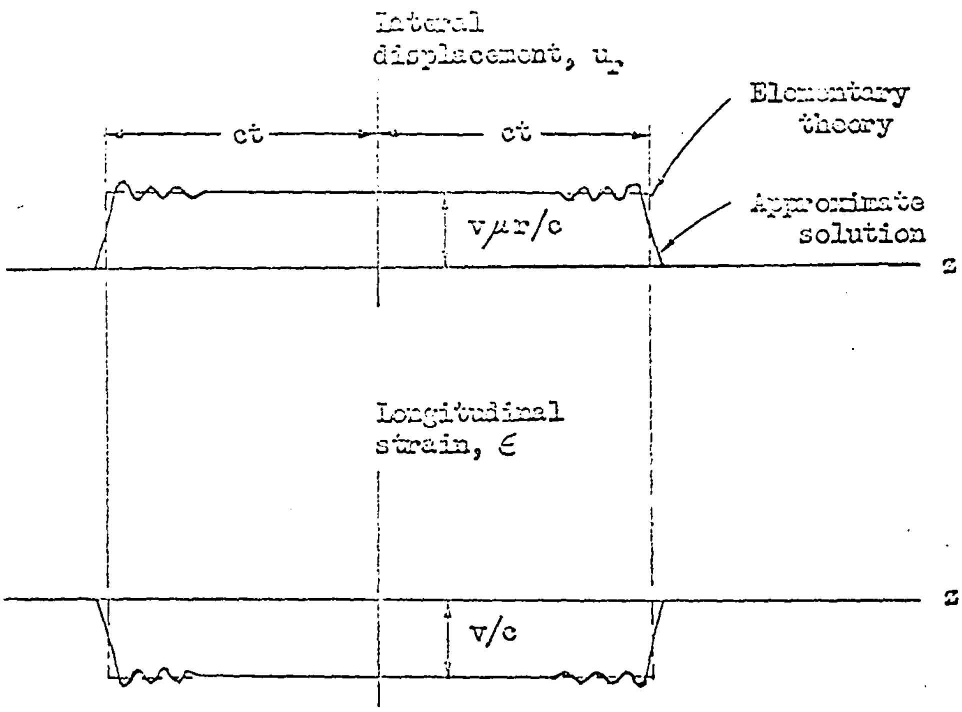


Fig. 3. Long time approximation of lateral displacement and longitudinal strain--from Skalak (10)

increasing length of the rod as the pulse propagates. The plot of the long time approximation of the lateral displacement and the axial strain are shown in Fig. 3.

Of significance in this figure is the overshoot of the elementary theory by the initial rise of the pulse form. This is attributed to radial inertia effects. There is also an oscillation of the strain in the rod behind the pulse front about the final value; this is not predicted by the elementary theory.

Griffith (11) presented in 1921 a theory describing the fracture of a brittle material such as glass, stone, or gray cast iron. Because the strain at which most brittle materials fracture is less than one per cent, he concluded that the applied force must be much less than the force which holds the atoms of material together. He pointed out, therefore, that the fracture of this material probably started at a crack which existed prior to the application of the stress. He explained his theory in this manner. Assume that the material contains a long crack of width $2a$ and that a stress S is applied at right angles to the crack (see Fig. 4). Because of the existence of the crack there is a decrease in the stored elastic energy of the material since there obviously is no material in the space where the crack is. Work is done, however, against surface tension in creating a new surface, and as the crack propagates, this type of energy is increasing. Thus, it follows that as cracking proceeds strain energy of the material is being converted to that energy associated with the formation of new surfaces.

The difference in the stored elastic energy and the value it would have had in the absence of the crack, Griffith found to be

$$\Delta U = - \frac{\pi S^2 a^2}{E} \qquad (1)$$

where S is the uniformly applied stress, a is one-half the crack width, and E is Young's modulus.

He found the work done against the surface tension T in the forming of the new surface to be

$$W = 4 a T \qquad (2)$$

Thus, the energy balance may be written

$$\frac{\partial}{\partial a} \left(- \frac{\pi S^2 a^2}{E} + 4 a T \right) = 0 \qquad (3)$$

Griffith concluded that the crack will spread if the energy in the bracket of Eq. 3 decreases with an increase of a, one-half the crack width. This would be true if

$$\frac{2 \pi S^2 a}{E} > 4 T \qquad (4)$$

According to Mott (12) there is another term that should be contained in the energy balance. This term would account for the kinetic energy of the material as the Griffith-type crack is propagated. From dimensional considerations Mott suggested that this term might be

$$KE = \frac{k \rho a^2 \dot{a}^2 S^2}{2 E^2} \qquad (5)$$

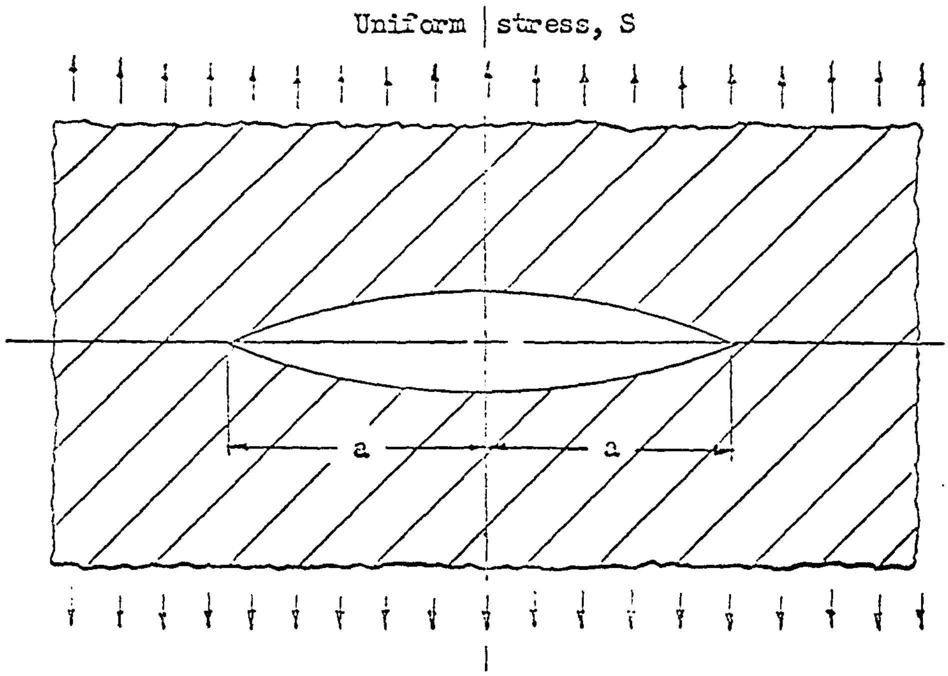


Fig. 4. Brittle crack propagation--from Griffith (11)

where k is an unspecified constant, ρ is the density, and $\dot{a}$ is the crack velocity. The energy balance in this case then becomes

$$\frac{\partial}{\partial a} \left(- \frac{\pi S^2 a^2}{E} + 4 a T + \frac{k \rho a^2 \dot{a}^2 S^2}{2 E^2} \right) = 0 \qquad (6)$$

If no energy is added by external forces during the crack propagation, the differential is zero as shown, and the crack propagation velocity is

$$\dot{a} = \left[\frac{2 \pi E}{k \rho} \left(1 - \frac{2 T E}{a S^2 \pi} \right) \right]^{\frac{1}{2}} \qquad (7)$$

The contribution of Roberts and Wells (13) was the determination of the kinetic energy of the material disturbed by the propagation of the crack. When $\dot{a}$ is zero, no kinetic energy is present, and

$$\frac{2 \pi S^2 a}{E} = 4 T \qquad (8)$$

These authors defined this value of a as one half the critical width. If it is denoted as a_o and solved for in the previous equation, the result is

$$a_o = \frac{2 T E}{\pi S^2} \qquad (9)$$

Upon the substitution of this value of a_o into Eq. 7, the equation becomes

$$\dot{a} = \left[\frac{2\pi E}{k\rho}\left(1 - \frac{a_o}{a}\right)\right]^{\frac{1}{2}} \qquad (10)$$

Whenever a is much larger than a_o , the limiting value of $\dot{a}$ is reached, and the value of the terminal velocity is

$$\dot{a}_T = \sqrt{\frac{2\pi}{k}}\sqrt{\frac{E}{\rho}} \qquad (11)$$

It appeared to Roberts and Wells that the kinetic energy must be limited to that volume of material which receives communication of any crack displacement and that this volume is limited by the velocity of the elastic wave. They determined the kinetic energy of the disturbed material and arrived at a value of the term $(2\pi/k)^{\frac{1}{2}}$ of 0.38. Their value for the velocity of crack propagation is therefore

$$\dot{a} = 0.38\,(1 - a_o/a)^{\frac{1}{2}}\,(E/\rho)^{\frac{1}{2}} \qquad (12)$$

This equation shows the terminal value of the velocity to be a definite fraction of the elastic wave velocity. Included in their report is a table showing the ratio of the terminal velocity of crack propagation to the elastic wave velocity as observed over a 15 year period by a number of investigators. The values of this ratio vary from 0.2 to 0.4.

The application of the Griffith theory to the fracture of Portland cement concrete was the object of an investigation by Kaplan (14). He determined the rate at which the strain energy of the material must be released in order that the crack be extended. Using concrete beams with simulated notches he determined experimentally the value of the rate of release of this energy. On comparing the theoretical value to the laboratory-determined one, he found the former to be on the average 20 per cent smaller.

A large amount of work has been done by investigators bent on determining the characteristics of fracture in glass. In his summary of these characteristics Shand (15) lists the following:

1. Fracture originates at smaller flaws already present.
2. Because of the concentrated stress at the crack tip, propagation begins at a minimum stress and is accelerated with the extension of the crack.
3. The process continues, although not in every fracture, until a limiting velocity is reached. This velocity is related to the velocity of elastic wave propagation in the glass.

Excellent photographs have been obtained by Christie (16) showing the fracture in glass and other materials by impact loads. Included in his article are a series of photographs commencing with the formation of the first crack after the impact, and progression of the fracture is portrayed until the material is completely fractured. The photographs show new cracks appearing in the material before the earlier-formed cracks have completely traversed the specimen. Christie ascribes this phenomenon to reflections of the elastic wave at material imperfections in its path. To an observer of the photographs a number of separately formed cracks seem to appear simultaneously and then to join to complete the fracture of the specimen.

The scabbing type of fracture has been investigated in materials of various ductilities by Broberg (17). Scabbing is a direct result of the so-called "Hopkinson effect," i. e., the reflection of a compression pulse at a free surface as a tension pulse. It may be identified by the material which is either cracked or spalled off in layers parallel to the free surface.

In his study of the scabbing fracture in a brittle material Broberg used granite as the resistant material. His method consisted of placing a high explosive charge on one side of a granite plate and detonating it. After the explosion he carefully studied the cracks formed on the side opposite the one where the explosion had taken place.

For the study of this type of fracture in a ductile material he used steel specimens in the arrangement shown in Fig. 5. The specimens varied in size from 3 to 24 mm in diameter. Multiple scabs were produced in most instances. A typical specimen reassembled after the detonation is shown in Fig. 6.

Foremost of the conclusions arrived at by Broberg was the necessity of including time as a factor when studying fracture as caused by impulsive loading. Some previous investigators had merely concluded that fracture would occur on a section when the stress there reached the static fracture stress. Broberg recognized that fracture can be produced only if the stress is applied for a sufficiently long time. No quantitative description of his results are included in his paper.

A method of determining the speed of propagation of cracks was developed by Hudson and Greenfield (18). Several nichrome wires were cemented to the surface of the specimen as shown in Fig. 7. The wires were parallel to themselves but were perpendicular to the path of the advancing crack. Only one crack existed, and its existence was insured

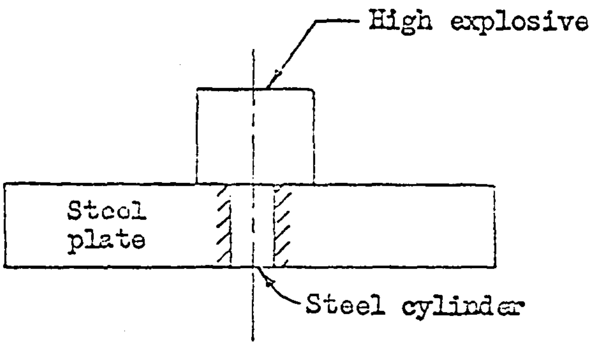


Fig. 5. Arrangement in experiment with steel--from Broberg (17)

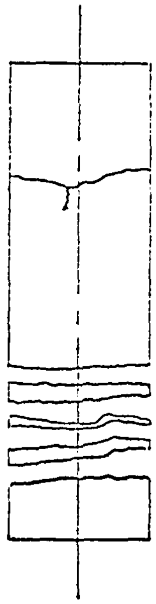


Fig. 6. Axial cross section through steel alloy cylinder after detonation of high explosive--from Broberg (17)

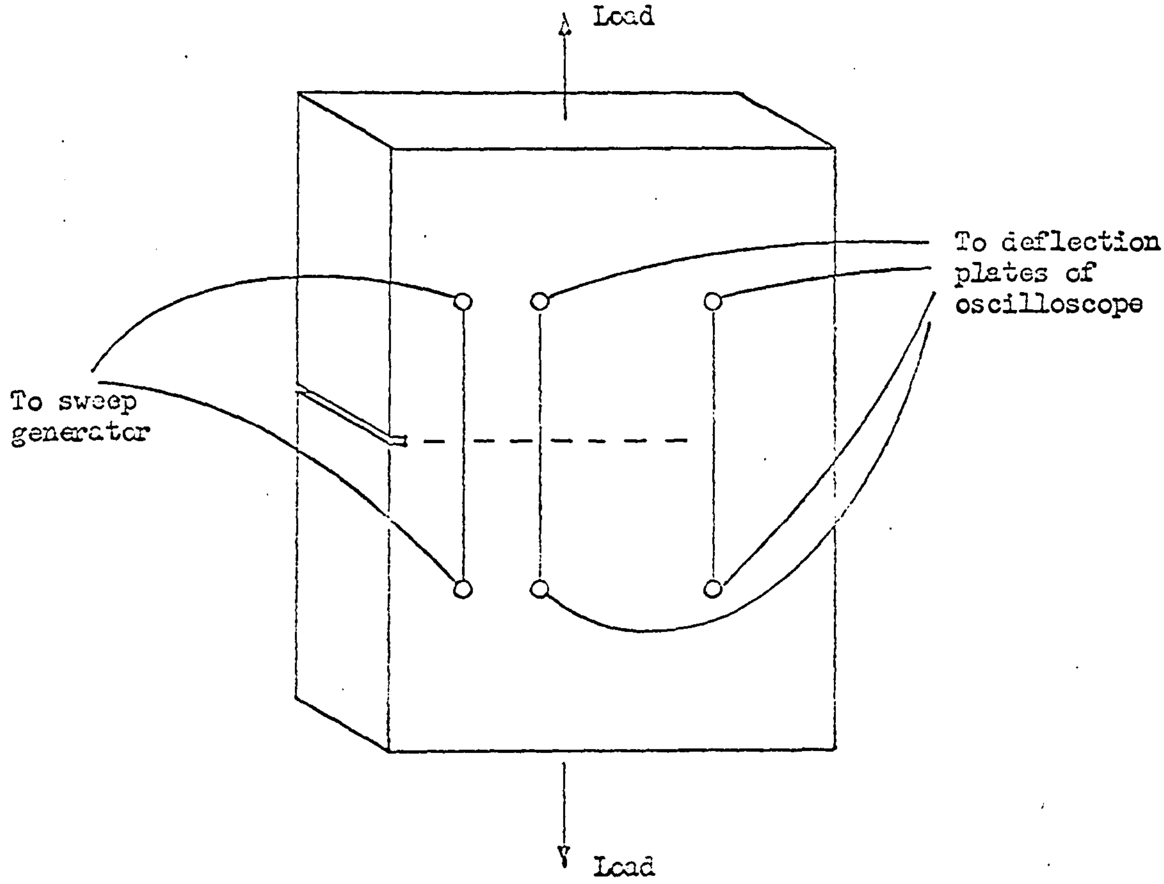


Fig. 7. Specimen for determining crack propagation velocity--from Hudson and Greenfield (18)

by providing the notch shown on the left side of the specimen. As the crack propagated, it broke successive wires, and this breakage produced voltage steps on the deflection plates of the oscilloscope. A standard oscillator provided a calibration wave for timing purposes, and the results as shown on the face of the oscilloscope were photographed.

Regarding longitudinal pulse propagation in a cylindrical rod of an elastic material, there is general agreement among the investigators as to certain aspects of the process. Several statements of this agreement are made below. In each case the pulse referred to is a "long" pulse.

1. The pulse will travel at the theoretical velocity and without distortion.
2. The strains will be uniform over the cross section of the rod and may be accurately determined by means of SR-4 strain gages located on the surface.
3. Strains may be accurately predicted by the elementary theory although the actual strains will overshoot and oscillate about the predicted values.

In the literature there appear to be a number of inconsistencies involving the characteristics of the crack propagation leading to fracture. Undoubtedly, the mechanism of fracture is considerably more complex than those concepts identified with the present state of knowledge. Statements expressing present agreement as to fracture in brittle materials are made below.

1. Whenever the applied stress is greater than the static tensile strength of the material, a crack will be propagated from an imperfection which was existent prior to the application of

the stress.

2. The propagation of the crack is accelerated until a limiting velocity is reached.

3. As the crack is extended, the stored elastic energy is converted to other forms of energy. Among the other forms are the energy associated with the formation of the new surface and the kinetic energy of the disturbed material.

THEORETICAL ANALYSIS

In this investigation an impact force is applied to one end of a long steel rod which is in contact with a plaster specimen. The character of the resulting impulse is determined, and the changes which result because of reflection and transmission at the interface between the steel rod and the plaster rod are investigated. The materials and their geometry have been selected so that the fronts of two tensile pulse meet at a desired cross section in the plaster specimen. When the intensities of the two pulses are superimposed, the tension is sufficient to cause transverse crack propagation in the brittle material. A plan for observing the nature of the fracture process is devised.

Velocity of Longitudinal Wave Propagation

The elastic theory of wave propagation indicates that wave velocities are dependent on the elastic constants and the density of the material. The exact theory of Pochhammer and Chree has been presented by Love (19). This theory is limited in its application, and its complex features introduce many difficulties in the analysis of the propagation of a pulse of a given form. Consequently, recourse is made to the approximate theories which in many cases yield surprisingly good results. In the investigations reported in this paper the elementary theory of Lord Rayleigh has been employed. Bancroft (20) has shown that whenever the rod diameter is less than one-tenth the wave length, the elementary theory approximates the exact theory to within 0.1 per cent.

Although a pulse was the nature of the disturbance in these investigations, the elementary wave theory will be analysed. Investigators (8, 21) agree that whenever the length of the pulse is equivalent to at least eight times the rod diameter, its velocity and intensity may be accurately predicted by means of the elementary theory.

Certain assumptions are made in developing the elementary wave theory. They are repeated here.

1. Plane sections remain plane.
2. Only uniform axial stresses exist on the cross section.
3. Radial inertia is neglected.

In Fig. 8 a differential element of a rod transmitting an elastic wave is shown. The equation of motion for the element is

$$\frac{\partial S}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{13}$$

Hooke's law, as applied in this case, is

$$S = E \frac{\partial u}{\partial x} \tag{14}$$

Combining Eqs. 13 and 14, the result is

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{E/\rho} \frac{\partial^2 u}{\partial t^2} \tag{15}$$

This is, of course, the well-known wave equation in which the velocity of wave propagation is

$$c = (E/\rho)^{\frac{1}{2}} \tag{16}$$

Intensity of Stress in a Longitudinal Pulse

Although the stress on a cross section and the pulse propagation velocity are independent of each other, the stress does depend upon the particle velocity. Consider a pulse of length L in which the stress is uniform and of intensity S and which is propagating along the rod of cross sectional area A.

The total energy of the pulse is

$$(S\ A)\ (\epsilon L) = \frac{S^2\ A\ L}{E} \tag{17}$$

where ϵ is the axial strain and E is Young's modulus.

This total energy consists of potential energy whose value is

$$PE = (\tfrac{1}{2}\ S\ A)\ (\epsilon L) = \frac{S^2\ A\ L}{2\ E} \tag{18}$$

and kinetic energy whose value is

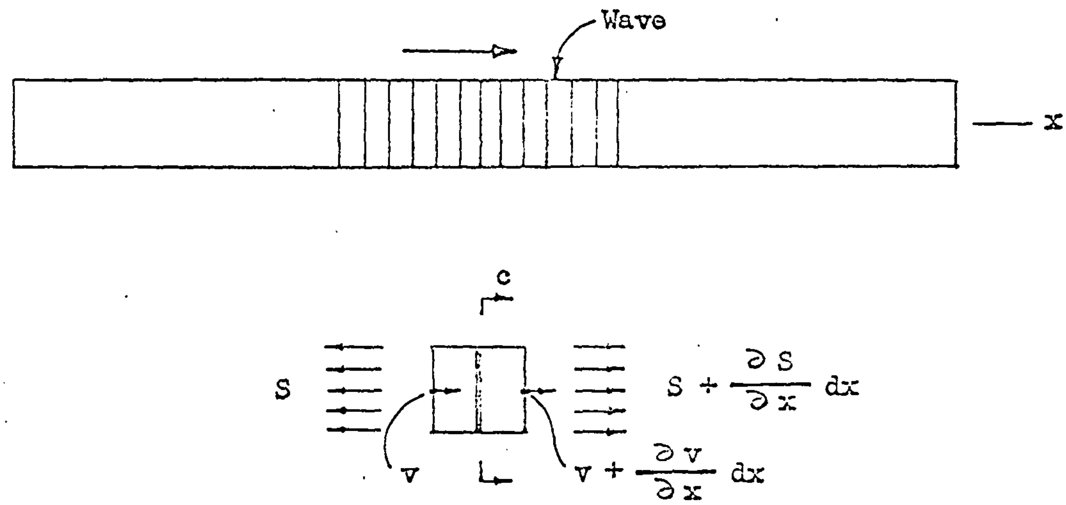
$$KE = \tfrac{1}{2}(\rho\ A\ L)\ \ v^2 \tag{19}$$

On equating the total energy to the sum of its parts, the stress is found to be

$$S = v\ \ (E\rho)^{\frac{1}{2}} \tag{20}$$

Stress Produced by Longitudinal Impact

In each experiment that was performed the pulse was produced by the impact of a moving rod on a second rod of the same material. In Fig. 9



In the above,

S = axial stress

v = particle velocity

c = wave propagation velocity

Fig. 8. Element of a rod transmitting an elastic wave

the moving rod A strikes the stationary rod B. A pulse moves in both directions away from the interface I which has a velocity v_I.

The equation of motion of a portion of both rods and containing the interface is

$$S_A - S_B = 0 \tag{21}$$

Since $S = v\,(E\rho)^{\frac{1}{2}}$, where v is the velocity of the particles in the zone of the pulse relative to the unstressed portion of the rod, on substituting into Eq. 20,

$$(v_A - v_I)\ (E_A\ \rho_A)^{\frac{1}{2}} - (v_I - v_B)\ (E_B\ \rho_B)^{\frac{1}{2}} = 0 \tag{22}$$

If

$$p = \left(\frac{E_B\ \rho_B}{E_A\ \rho_A}\right)^{\frac{1}{2}}, \tag{23}$$

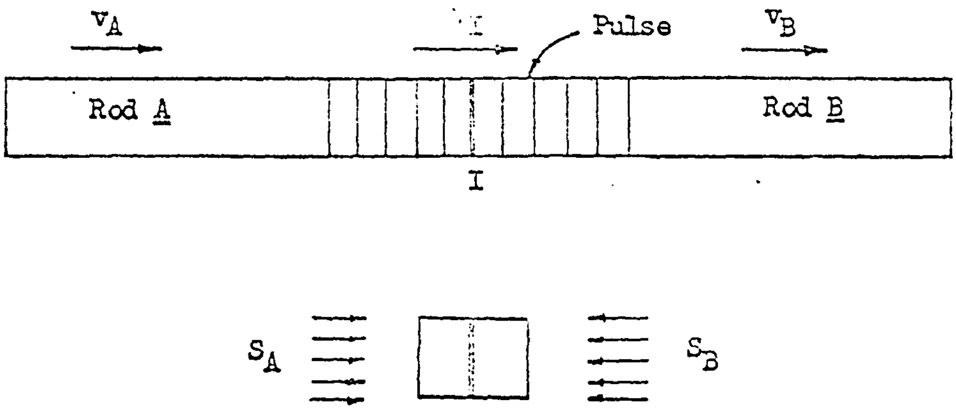
then Eq. 22 may be solved for the velocity of the interface. The result is

$$v_I = \frac{v_A + v_B\ p}{p + 1} \tag{24}$$

On substituting Eq. 24 into Eq. 20,

$$S_A = (v_A - v_I)(E_A\ \rho_A)^{\frac{1}{2}} = \frac{(v_A - v_B)\ p}{p + 1}\ (E_A\ \rho_A)^{\frac{1}{2}} \tag{25}$$

If rod B is stationary, and if both rods are of the same material,



In the above,

S = axial compressive stress

v = velocity

Fig. 9. Stress caused by impact of one rod on a second one.

the stress in the pulse moving away from the interface is

$$S = \frac{v_A}{2} (E_A \rho_A)^{\frac{1}{2}} \qquad (26)$$

Pulse Transmission and Reflection at a Boundary

Of particular interest in this investigation are the changes effected on a compressive pulse as it encounters a boundary between two materials or as it encounters a free surface. In Fig. 10 the primary pulse of stress intensity $\underline{S}$ and particle velocity $\underline{v}$ approaches the boundary between rod $\underline{A}$ and rod $\underline{B}$. A portion of the pulse will be reflected from the boundary, and a portion will be transmitted through it. The compressive stress in the reflected pulse will be denoted $\underline{S_r}$, and the particle velocity associated with it will be denoted $\underline{v_r}$. Correspondingly, these symbols for the transmitted pulse will be $\underline{S_t}$ and $\underline{v_t}$.

The equation of motion of the differential element containing portions of both rods including the boundary is

$$(S + S_r) - S_t = 0 \qquad (27)$$

From Eq. 20 the particle velocity to the left of the boundary is

$$v - v_r = \frac{S - S_r}{(E_A \rho_A)^{\frac{1}{2}}} \qquad (28)$$

and that just to the right of it is

$$v_t = \frac{S_t}{(E_B \rho_B)^{\frac{1}{2}}} \qquad (29)$$

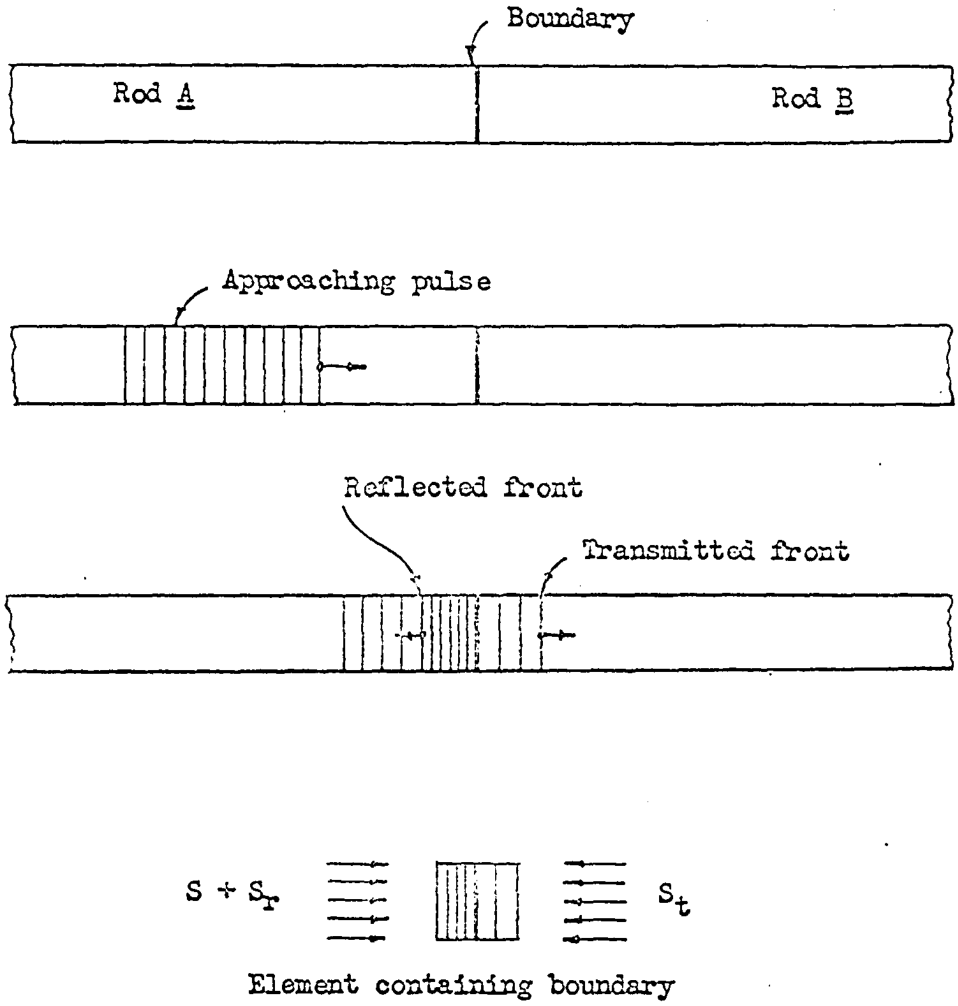


Fig. 10. A compressive pulse traversing the boundary between two rods

But, these velocities are equal. Equating them, and if

$$p = \left(\frac{E_B \quad \rho_B}{E_A \quad \rho_A} \right)^{\frac{1}{2}} \qquad (23)$$

the stress relationship becomes

$$S_t = p \quad (S - S_r) \qquad (30)$$

When Eqs. 27 and 30 are combined, the result is

$$S_r = \frac{(p - 1)}{(p + 1)} \; S \qquad (31)$$

$$S_t = \frac{2p}{(p + 1)} \; S \qquad (32)$$

For a pulse passing from steel to plaster, p has a value of 0.09. The reflected stress has a value of -0.83 S, the negative sign indicating a tensile stress if the primary stress is compressive. The transmitted stress has a value of 0.17 S, indicating that this stress is compressive if the primary stress is compressive.

There is one other observation of this particular analysis upon which a comment should be made. For a pulse approaching the free end of a rod, the value of p is essentially zero. By the use of Eqs. 31 and 32, the stress in the reflected and the transmitted pulses may be determined. In this case the stress in the reflected pulse is -S which may be interpreted that a compressive pulse is reflected as tensile but without loss in magnitude. The magnitude of the transmitted pulse is zero.

Velocity of Transverse Crack Propagation

If the applied stress on the cross section of the rod is only slightly larger than the static fracture stress, one or more of the microcracks which are present in the brittle material will begin to grow and will eventually traverse the cross section. For the plaster used in this research the critical width of the crack is quite small, on the order of $(10)^{-3}$ in. for a static fracture stress of 500 psi. This would mean that when such a crack had grown to $(10)^{-1}$ in., the velocity of crack propagation would be over 99 per cent of the terminal velocity.

For the purpose of evaluating the velocity of transverse crack propagation, the following assumptions are made.

1. Only one crack is propagated. It originates at point $\underline{0}$ as shown in Fig. 11.
2. The crack propagates at the terminal velocity $\underline{\dot{a}}_T$, and the crack tip is propagated with a circular front, passing the stations $\underline{1}$, $\underline{2}$, $\underline{3}$, and $\underline{4}$ on the cross section of the rod in Fig. 11.

Let $\underline{d}$ with subscripts be the distance between points on the cross section as noted by the subscripts; thus, $\underline{d}_{01}$ represents the distance between $\underline{0}$ and $\underline{1}$.

In a similar manner $\underline{t}$ with subscripts is the time for the crack front to move between points of subscripts; thus, $\underline{t}_{12}$ represents the time for the front to move from point $\underline{1}$ to point $\underline{2}$.

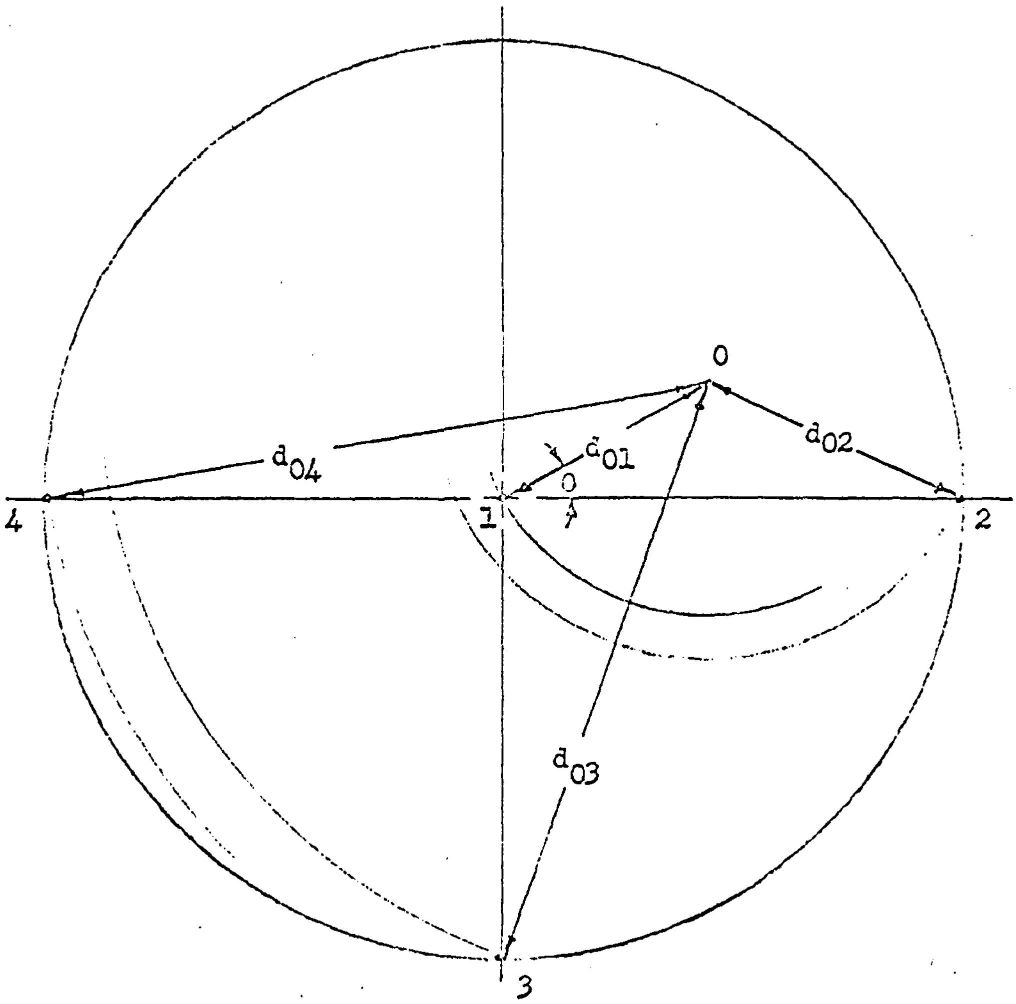


Fig. 11. Transverse crack propagation in a slender rod

On the basis of the foregoing assumptions,

$$t_{12} = \frac{(d_{02} - d_{01})}{\dot{a}_T} \tag{33}$$

$$t_{13} = \frac{(d_{03} - d_{01})}{\dot{a}_T} \tag{34}$$

$$t_{14} = \frac{(d_{04} - d_{01})}{\dot{a}_T} \tag{35}$$

The distances may be represented in these equalities.

$$d_{02} = \left[(d_{12} - d_{01} \cos \theta)^2 + (d_{01} \sin \theta)^2\right]^{\frac{1}{2}} = (d_{12}^2 - 2\, d_{12}\, d_{01} \cos \theta + d_{01}^2)^{\frac{1}{2}} \tag{36}$$

$$d_{03} = \left[(d_{01} \cos \theta)^2 + (d_{13} + d_{01} \sin \theta)^2\right]^{\frac{1}{2}} = (d_{13}^2 + 2\, d_{13}\, d_{01} \sin \theta + d_{01}^2)^{\frac{1}{2}} \tag{37}$$

$$d_{04} = \left[(d_{14} + d_{01} \cos \theta)^2 + (d_{01} \sin \theta)^2\right]^{\frac{1}{2}} = (d_{14}^2 + 2\, d_{14}\, d_{01} \cos \theta + d_{01}^2)^{\frac{1}{2}} \tag{38}$$

When the distances are substituted into Eqs. 33, 34, and 35, these results are obtained.

$$t_{12} = \frac{(d_{12}^2 - 2\, d_{12}\, d_{01} \cos \theta + d_{01}^2)^{\frac{1}{2}} - d_{01}}{\dot{a}_T} \tag{39}$$

$$t_{13} = \frac{(d_{13}^2 + 2\ d_{13}\ d_{01}\ \sin\theta + d_{01}^2)^{\frac{1}{2}} - d_{01}}{\dot{a}_T} \qquad (40)$$

$$t_{14} = \frac{(d_{14}^2 + 2\ d_{14}\ d_{01}\ \cos\theta + d_{01}^2)^{\frac{1}{2}} - d_{01}}{\dot{a}_T} \qquad (41)$$

If Eq. 39 is divided by Eq. 40, the result is

$$\frac{t_{12}}{t_{13}} = \frac{(d_{12}^2 - 2\ d_{12}\ d_{01}\ \cos\theta + d_{01}^2)^{\frac{1}{2}} - d_{01}}{(d_{13}^2 + 2\ d_{13}\ d_{01}\ \sin\theta + d_{01}^2)^{\frac{1}{2}} - d_{01}} \qquad (42)$$

If Eq. 39 is divided by Eq. 41, the result is

$$\frac{t_{12}}{t_{14}} = \frac{(d_{12}^2 - 2\ d_{12}\ d_{01}\ \cos\theta + d_{01}^2)^{\frac{1}{2}} - d_{01}}{(d_{14}^2 + 2\ d_{14}\ d_{01}\ \cos\theta + d_{01}^2)^{\frac{1}{2}} - d_{01}} \qquad (43)$$

All of the time quantities in Eqs. 42 and 43 are measureable as are all the distances except $\underline{d}_{01}$. The only unknown quantities, therefore, are $\underline{d}_{01}$ and $\underline{\theta}$. When these equations are solved for these two unknown quantities, the origin of the crack becomes known. Substitution of $\underline{d}_{01}$ and $\underline{\theta}$ into any of the Eqs. 39, 40, or 41 will yield the crack propagation velocity $\underline{\dot{a}}_T$.

EXPERIMENTAL PROCEDURE

The apparatus used in conducting the experiment can best be understood by studying the schematic arrangement in Fig. 12. The slender rod labeled "Striker" was of steel and was propelled by air pressure through a tube. It struck a second steel rod labeled "Anvil" which was stationary. Both rods were 1½ in. in diameter, and the lengths of the striker and the anvil were 18½ in. and 35 in., respectively. These lengths were chosen after analytical considerations showed that they would produce in the specimen a tensile loading lasting approximately 200 microseconds.

When contact with the anvil was made by the striker, the capacitance in the "Sweep Initiation Circuit" was discharged. The voltage was sufficient to produce the triggering of the time sweep on the oscilloscope. The sweep period could be regulated on the oscilloscope, and since all subsequent events occurred in the 900 microseconds following the impact, this rough setting was employed. More precise measurements were required than those which could be had from the oscilloscope setting alone, so a 20 kilocycle per sec. sine wave was generated by an oscillator. The trace of this calibration wave also appeared on the oscillogram subsequently produced.

As the pulse passed along the anvil, its magnitude was measured by the use of longitudinally oriented resistance wire strain gages. In this case two gages were diametrically located so that any bending effects that might have existed were eliminated from the measurements. These SR-4

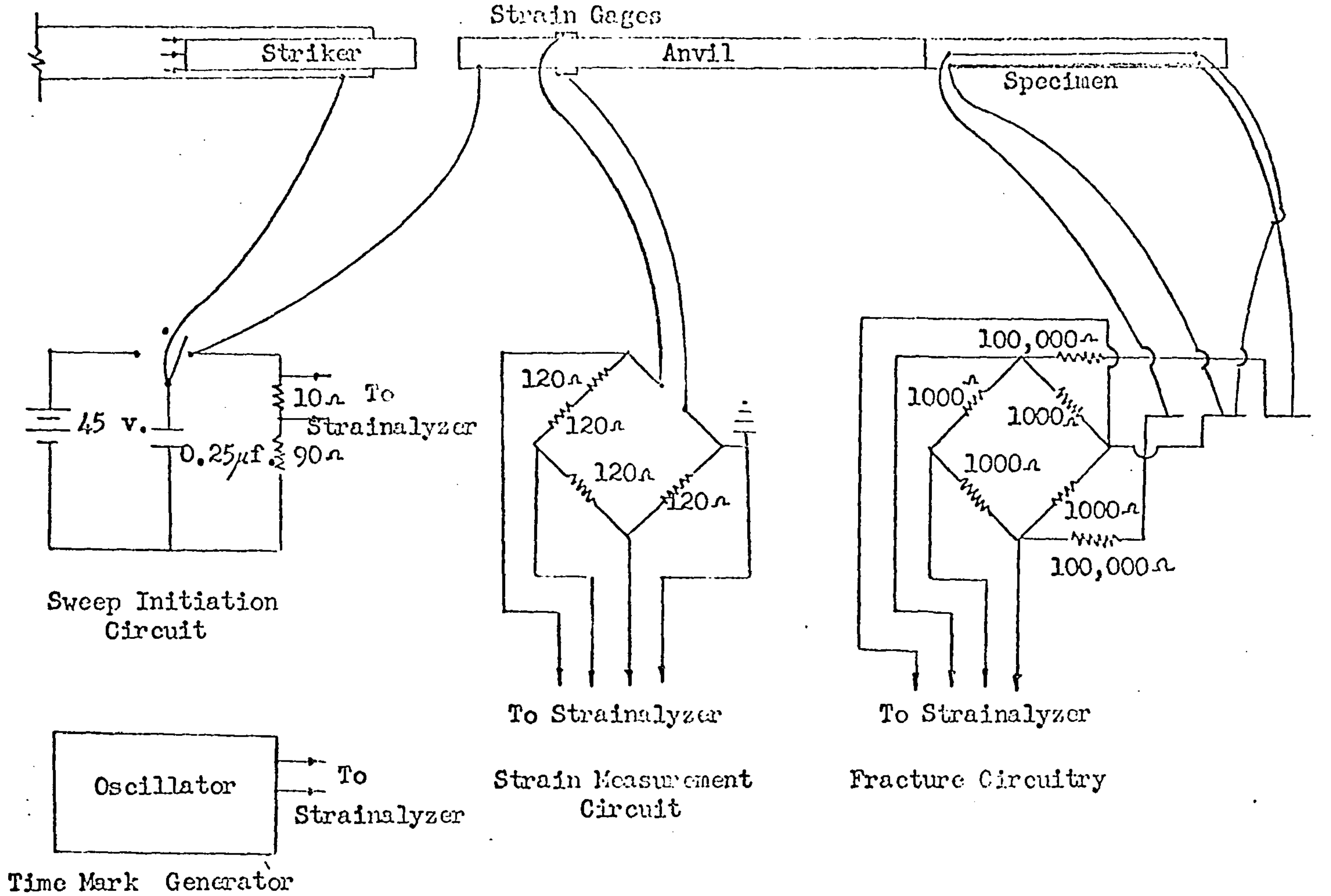


Fig. 12. Schematic arrangement of the apparatus

gages were incorporated in the bridge labeled "Strain Measurement Circuit", and the signal from this bridge was applied to the deflection plates of one channel of the oscilloscope.

The pulse traveling along the anvil was partially transmitted and partially reflected at the interface between the steel anvil and the plaster specimen. The magnitude of the transmitted pulse was measured in the same manner as was the pulse measured in the anvil. After having verified the transmissibility factor (steel to plaster) by a number of observations, in subsequent tests the stress in the plaster rod was determined solely by having a knowledge of the transmissibility factor.

In order to study the transverse crack propagation in the specimen, circuitry such as that labeled "Fracture Circuitry" in Fig. 12 was installed. Thin longitudinal lines of conducting silver paint were painted on the specimen. When the specimen fractured on any cross section, the breakage of the conductor unbalanced the bridge, and the resulting signal caused a movement of the trace on the oscilloscope. As many as four of the conductive paint circuits could be observed with the equipment available.

In a second series of tests information as to the time of completed fracture of the specimen was desired. In these cases the entire external surface of the rod in the vicinity of the expected break was coated with the conductive paint. This surface coat constituted the only fracture circuit.

A photograph of the arrangement of the apparatus may be seen in Fig. 13. Through the hose on the floor the compressed air was conveyed which provided the pressure for propelling the striker. The air was confined in the plenum chamber until the hand-operated valve was opened. Atop the plenum chamber was a gage for determining the pressure in it. This pressure determined the speed at which the striker was propelled, and the gage was used as control device for the test. At the extreme left of the photograph may be seen the steel anvil in contact with the white plaster rod. After the rod was shattered by the impact, the broken pieces were retrieved and fitted together for further observation.

The principal recording device employed in the laboratory experimentation was an Electron Tube Corporation H-42 Strainalyzer. In essence, the Strainalyzer is a four-channel oscilloscope that possesses bridges and calibration circuitry for use in measuring strains detected by the resistance gages. A Polaroid camera for photographing the results was attached to the Strainalyzer. The upper photograph of Fig. 14 shows the Strainalyzer located alongside the anvil and a specimen; the lower photograph of the figure is a closer view of the anvil and a specimen as they are mated and held in place by rubber clamps.

The sweep initiation circuit performed adequately, the sweep being initiated sufficiently early to record the arrival of the front of the pulse as it encountered the strain gages on the anvil. A sine wave was applied to one of the channels of the oscilloscope in order to insure accurate time measurements. This wave was generated by a Hewlett-Packard Model 202C oscillator whose accuracy had been previously confirmed.

The proper gage length of the strain gages was an item of consideration. As has been pointed out by Murphy _et al_. (22) the duration of the signal

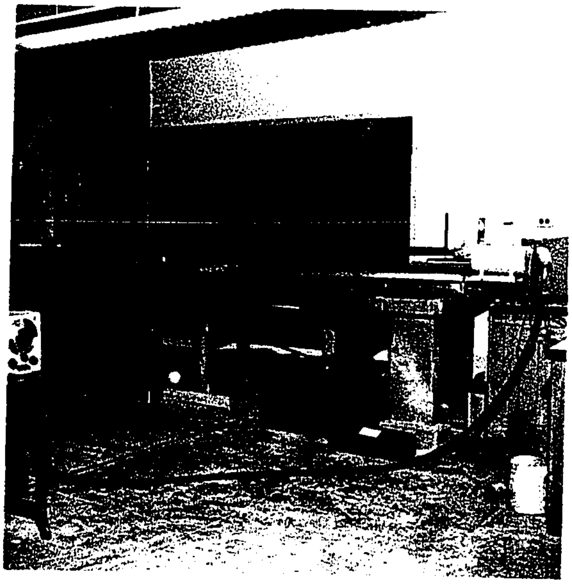


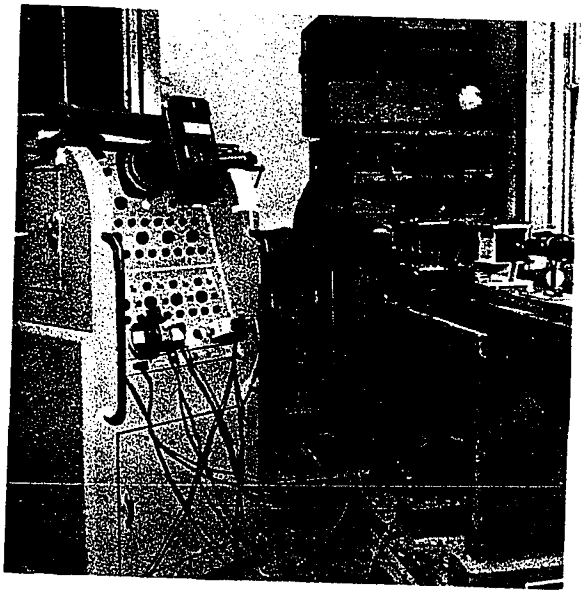
Fig. 13. Photograph of the arrangement of the apparatus

from this type of strain gage is the sum of the duration of the actual pulse plus the ratio of the gage length to the pulse propagation velocity. This would indicate that a short gage length were desirable. Also, the time for the signal of the pulse to rise to its maximum value, the so-called rise time, would be affected by an extremely long gage length. For these reasons and because of the ease in attaching them to the surface of the rod, SR-4 gages having a one-eighth in. gage length (type A-8) were employed.

The calibration circuits which were an integral part of the Strain-alyzer were quite beneficial in determing pulse intensities. This equipment had been calibrated by personnel in the Theoretical and Applied Mechanics Laboratory and had been found to be correct to within about one per cent.

The ends of all the rods were ground to a flat face. To insure continuity of material across the interface between the anvil and the specimen, a thin coating of cup grease was provided there.

The factors relating the transmission and reflection of the primary pulse at the interface between the anvil and the specimen was an object of the laboratory investigation. Since the result of each test was the destruction of the specimen and thereupon the loss of the attached strain gages, it was hoped that precise values of these factors could be established in the laboratory. In the Theoretical Analysis the values for the transmissibility factor and the reflectability factor have been shown to be 17 per cent and 83 per cent, respectively. As the laboratory investigation confirmed these values experimentally, no gages were installed on the specimens thereafter.



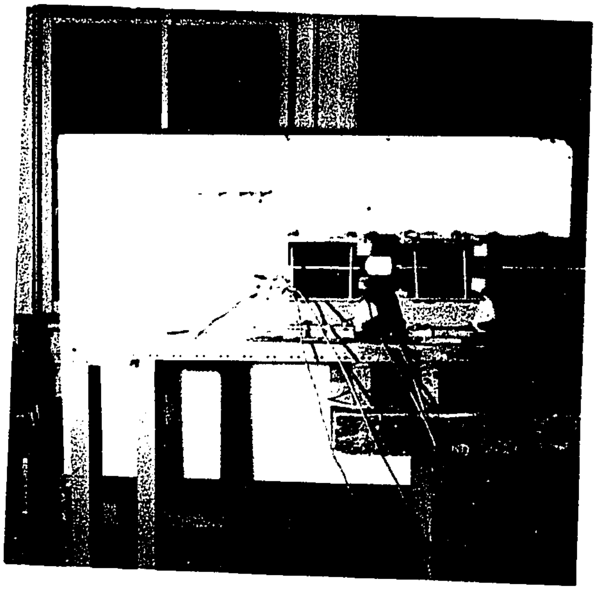


Fig. 14. Photographs of the recording equipment

In one series of tests four fracture circuits were incorporated on or within the specimen. Observing the specimen from the rear (i. e., looking at the end which would be mated with the anvil), these circuits were located and numbered as shown in Fig. 15.

Circuit No. 1 was installed by coating with silver conducting paint a 1/8 in. diameter hole which had been cast in the specimen at the time the plaster had been cast. At a distance of one in. from each end a radial hole was drilled so that the circuit might be brought out the sides of the specimen rather than through the ends. After the circuit was found to be complete, all holes were filled with plaster of the same type as that of the rod.

Circuits No. 2, 3, and 4 were strips of conducting silver paint of 1/8 in. width.

Fracture of the circuits unbalanced the bridge of which they were a part, and the unbalance was noted by a movement of the trace on the face of the oscilloscope. Two circuits were incorporated in one bridge; whenever one circuit was broken, the trace either moved up or down, and when the other circuit incorporated in that bridge was broken, the trace moved in a direction opposite to that of its first movement.

The center trace of the oscillogram shown in Fig. 16 typifies the action just described. The fracture of one circuit has caused the downward movement of the trace. Upon the breaking of the other circuit incorporated in the bridge, the trace has moved upward. The time at which each of the fractures occurred can be noted by the use of one of the sine curves which appear at the bottom of the oscillogram.

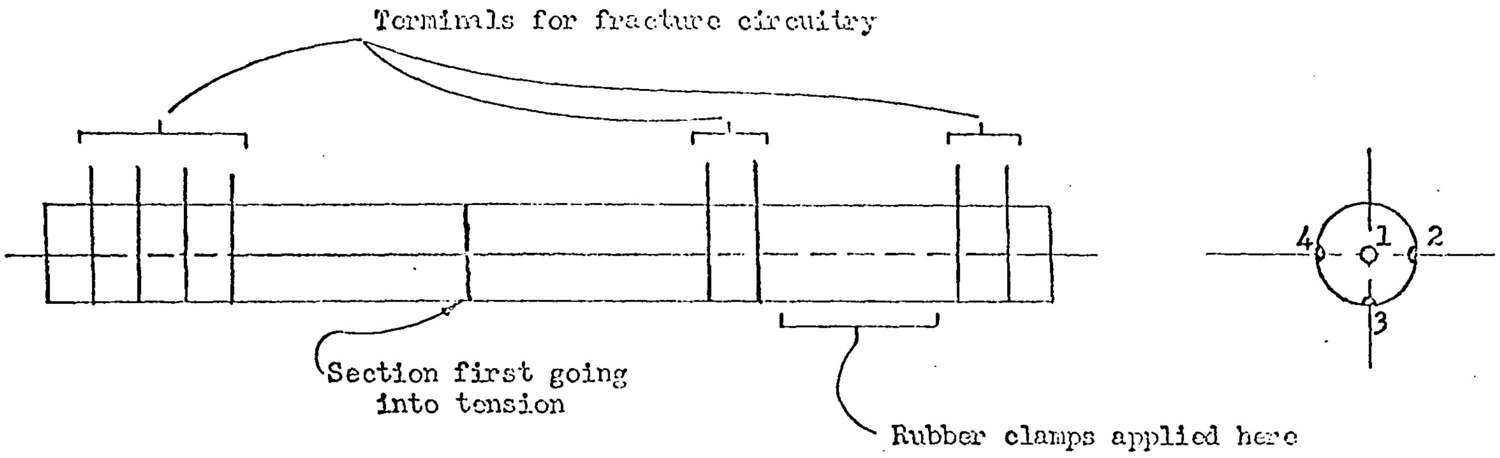


Fig. 15. Plaster specimen showing the location of four fracture circuits

In a third series of tests it was desired that the time of completed fracture of the cross section be determined. For these tests the entire surface of the specimen was coated with the conductive paint for a distance of four in. in both directions from that section where the break was expected to occur. The fracture circuitry in this case thus consisted of only a single circuit. When fracture was completed, the single circuit was broken, and the event was noted by the movement of the trace on the oscilloscope. Breakage times may be noted on the sine curves.

The plaster of which the rods were cast was a gypsum product marketed by the United States Gypsum Company. In every case the mixture was 0.45 lb of water per lb of plaster. Mixing of the dry plaster and water was accomplished manually, and when completed, the mixture was cast into paper molds. After the plaster had set, the molds were removed, and the ends were ground flat. The prepared specimens were 1½ in. in diameter and 24 in. in length.

Whenever the plaster rods were made, various test specimens were also prepared so that the mechanical properties and other characteristics of the plaster might be known. In order to observe the brittle nature of the plaster a tensile test was performed. A section of a one in. diameter rod was turned down in a lathe to a diameter of 0.505 in., and this specimen was dead-weight loaded until failure occurred. The curve plotted from the data obtained in this test is shown in Fig. 17.

The strength of this particular specimen was not nearly as great as that obtained using the standard briquettes in a tensile test. Observations made during this test revealed that some bending occurred and

Fig. 16. Typical oscillogram showing circuit breakage

that this accounted for the lower tensile strength of this specimen.

The properties in Table 1 were determined in the following manner. The compressive strength of the plaster was determined in conformance with ASTM Designation C 472-61 (23), except that the water-plaster ratio was 0.45 by weight. Tensile strength of the plaster was determined in conformance with ASTM Designation C 190-59 (24). The dynamic modulus of elasticity of the plaster was determined in conformance with ASTM Designation C 215-60 (25). Densities were calculated after carefully weighing and obtaining the lineal dimensions of the various test specimens. Pulse propagation velocities were determined from observations on the rods themselves as detected by the oscilloscope.

The material closely allied to the plaster in the rods was the steel of which the striker and the anvil were made. The pulse propagation velocity in the steel was determined by observing a pulse as it traveled back and forth along the system. The density was found from measurements made on the striker and the anvil. Young's modulus for the steel was determined by solving for it in the equation for the theoretical velocity, $c = (E/\rho)^{\frac{1}{2}}$. Properties of the steel which were applicable to this investigation are also found in Table 1.

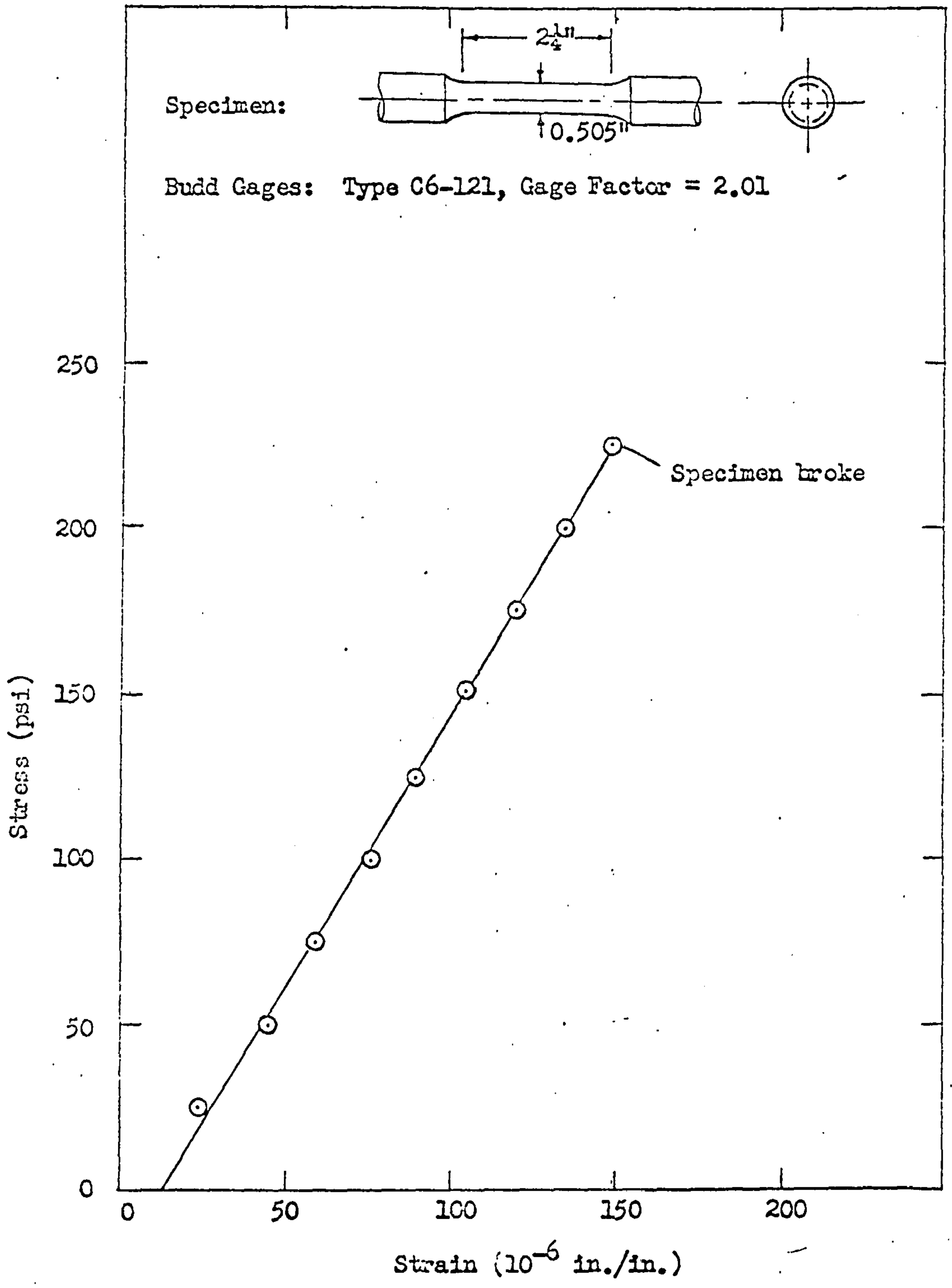


Fig. 17. Tensile stress-strain curve

Table 1. Material properties

Plaster

Dynamic modulus of elasticity, psi	Water/plaster, lb/lb	Compressive strength, psi	Tensile strength, psi	Density lb sec^2 $in.^{-4}$	Propagation velocity, in. $sec.^{-1}$
$1.30(10)^6$	0.45	3,650	375	$1.30(10)^{-4}$	100,000

Steel

Dynamic modulus of elasticity, psi	Water/plaster, lb/lb	Compressive strength, psi	Tensile strength, psi	Density lb sec^2 $in.^{-4}$	Propagation velocity, in. $sec.^{-1}$
$25.8(10)^6$				$7.36(10)^{-4}$	187,000

EXPERIMENTAL RESULTS

The laboratory investigations consisted of three separate sets of experiments. In the first set it was desired to verify pulse propagation velocities and the factors relating the intensities of the transmitted and reflected pulses at a boundary to the intensity of the primary pulse. In the second set of investigations it was desired to determine the origin of the crack which caused fracture of the rod, and to determine, if possible, its propagation velocity. After observing that several cracks originated almost simultanelusly at various points on the cross section of the rod, a third set of experiments were run in order to determine the time for complete fracture of the specimen.

A graphical history was prepared of the pulse propagation along the system from the instant of impact until the time that a section of the plaster rod first went into tension. In representing the history propagation velocities of 187,000 in. per sec. and 100,000 in. per sec. were used for the steel and the plaster, respectively. The intensity of the pulse created by the impact of the striker on the anvil is dependent upon the velocity of the striker at that instant, but for this history an initial pulse intensity of one psi compression was assumed.

The expected history of pulse propagation is shown in Fig. 18. The particular times that were chosen were done so because at that instant a pulse front is passing either a strain gage located on the anvil or on the specimen. Time commences with the impact of the striker on the anvil. All magnitudes of transmitted and reflected pulses are based on the assumed intensity of the initial pulse caused by the impact. It may be noted that

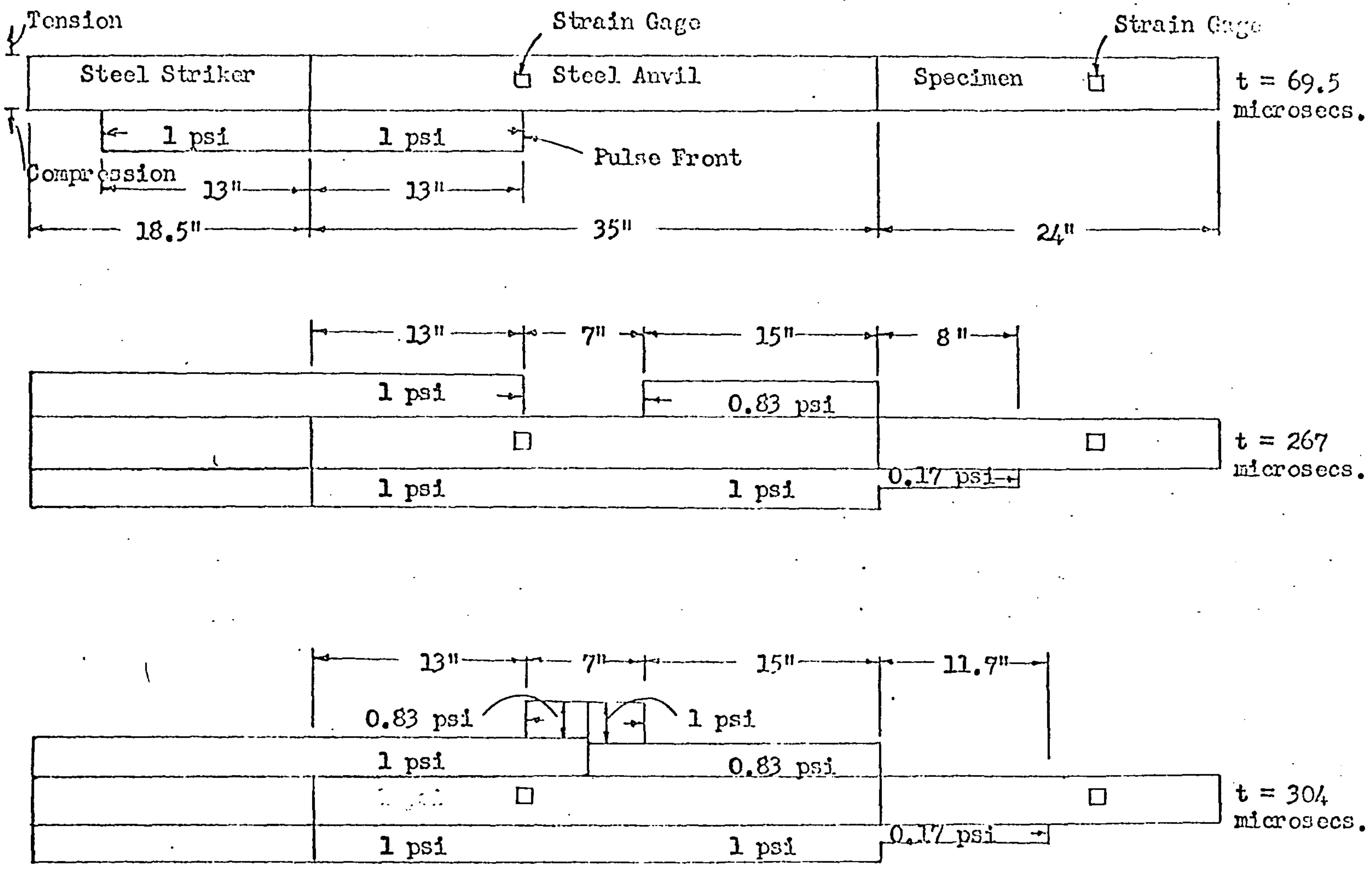


Fig. 18. Expected stress history of pulse propagation along the system (see appendix)

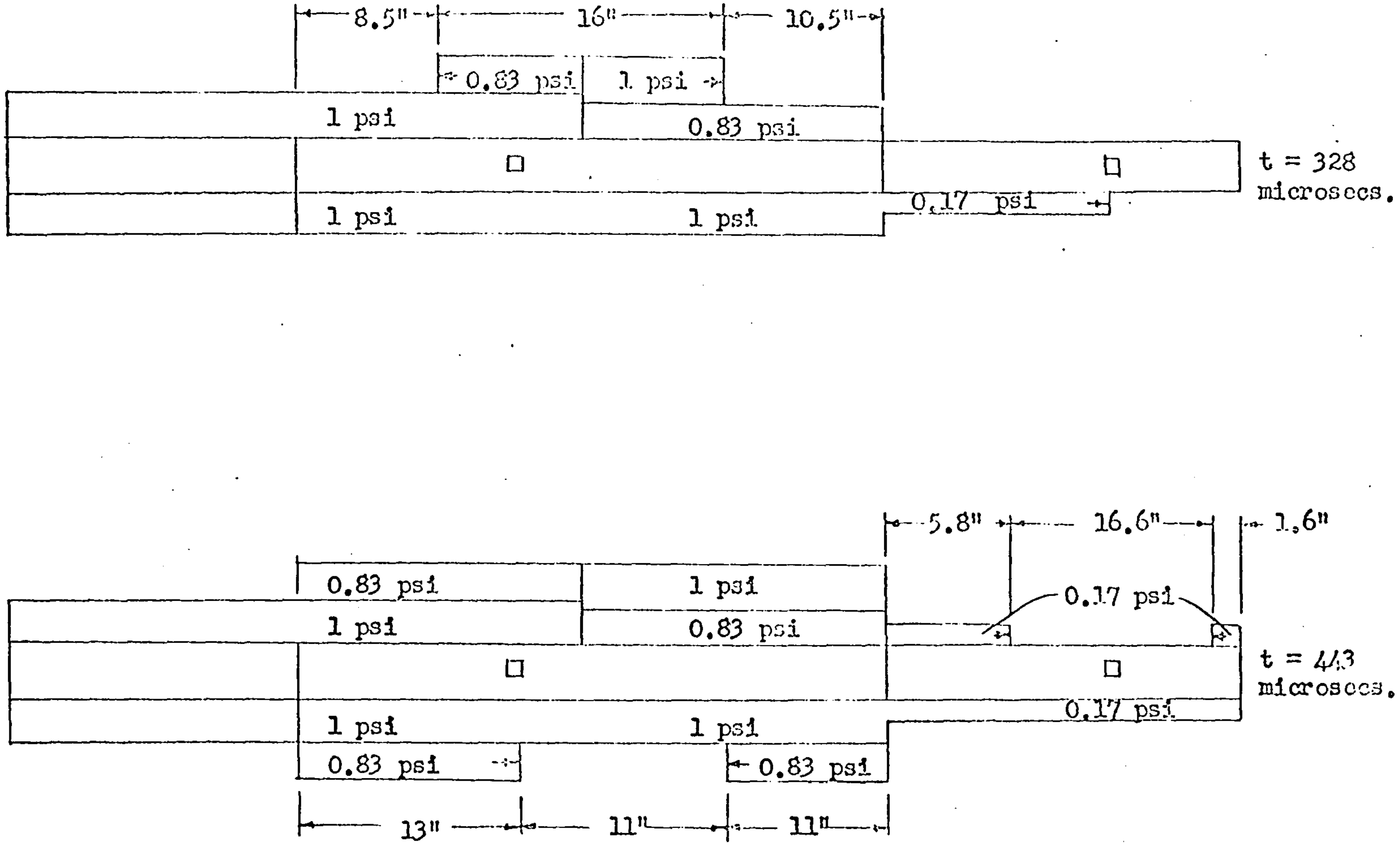


Fig. 18. (Cont'd) Expected stress history of pulse propagation along the system

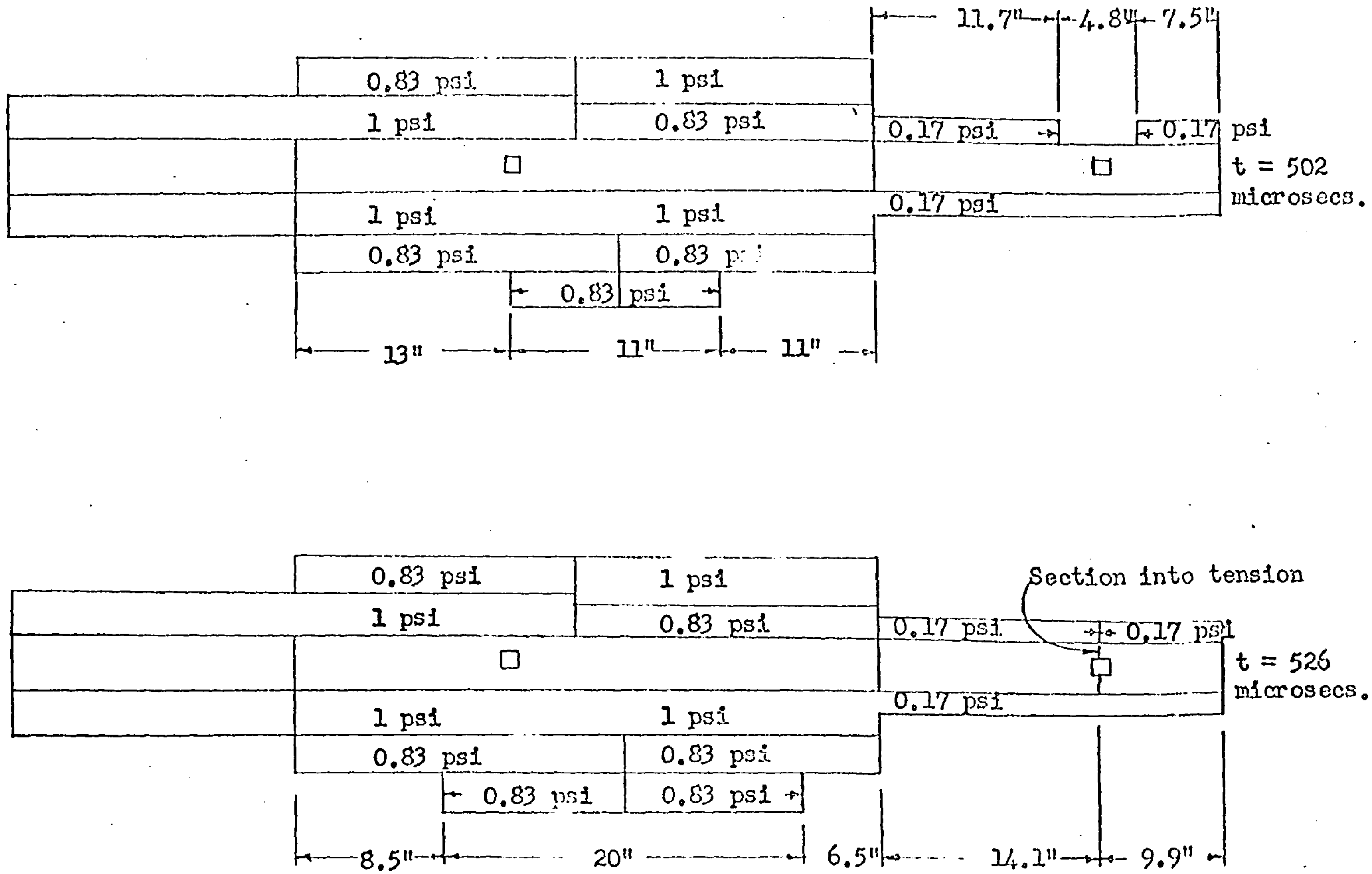


Fig. 18. (Con'd) Expected stress history of pulse propagation along the system

of the initial pulse, 83 per cent is reflected and 17 per cent is transmitted at the interface (boundary) between the anvil and the specimen. These are the theoretical factors as determined in the Theoretical Analysis. It may also be noted that after 526 microseconds the section of plaster located 9.9 in. from the free end goes into tension for the first time. It will remain in tension for the next 198 microseconds if fracture does not occur during that time.

Part of the information contained in this figure can be portrayed in a stress-time plot of the activity experienced by the strain gages. Fig. 19 is such a plot; it portrays the information that might be expected to be relayed by the strain gages located at those positions. It should be understood that this plot is theoretical in that it is based on the elementary theory. Some of the theoretical aspects may be commented upon here. The time for the pulse to rise to its full value is instantaneous, and the intensity remains at a constant value until it drops instantaneously to zero.

A laboratory investigation of this history was successfully accomplished a number of times. The strain gages were located on the anvil and on the plaster specimen in the same places as those shown on the figure of the expected stress history. A typical oscillogram of these results is shown in Fig. 20. The lower trace is the strain record as measured by the gage on the steel anvil; here compression is recorded in the downward direction. The middle trace is the calibration sine wave of 20 kilocycles per sec., and thus each of the cycles represents 50 microseconds. The upper trace is the transmitted pulse as recorded by the gage located on the plaster specimen. This gage was connected to the arm of the

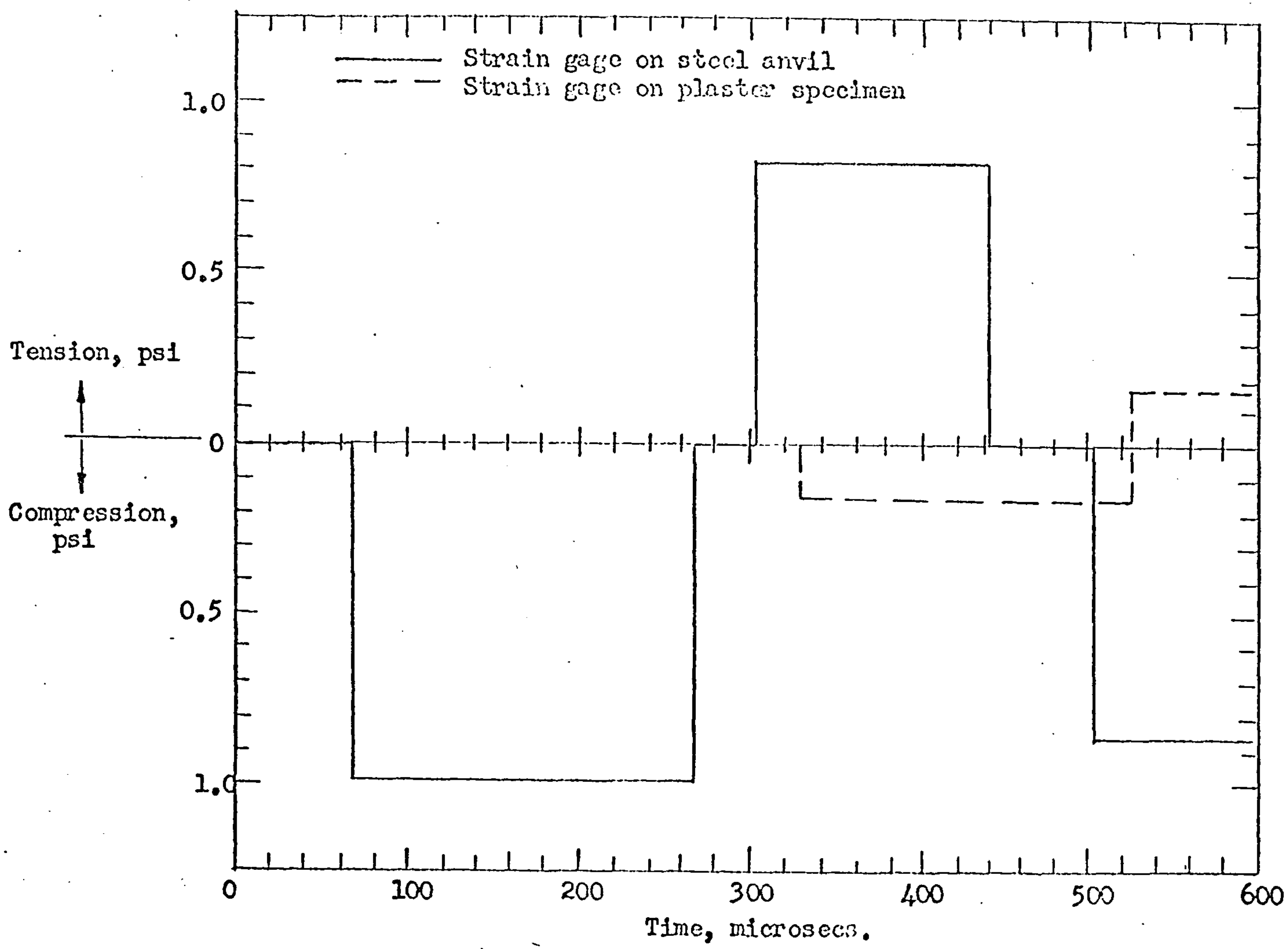


Fig. 19. Expected stress history at strain gage locations

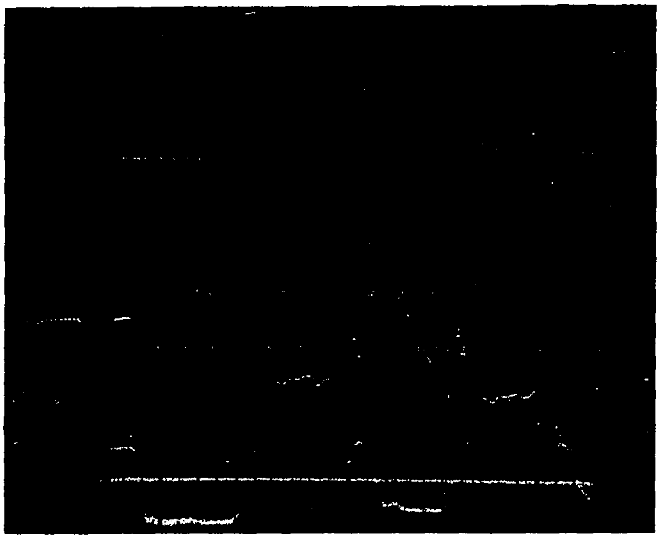


Fig. 20. History of stress at gage locations. (Compare with Fig. 19)

bridge which caused compression in this case to be recorded upward.

A comparison made between this oscillogram and the expected history of Fig. 19 reveals a close agreement. The rise times on the oscillogram are not instantaneous as are those shown on the expected history. A slight oscillation about the average value of the pulse intensity occurs on the oscillogram as might be expected. Since the scale factors for the pulse intensities of the two traces are different, the relative magnitudes of the two traces cannot be compared visually. The magnitudes of the several pulses passing a gage can be compared, however, as can the times that the various events occur.

For instance, the phase relationships of the two traces are as were expected. The magnitude of the second pulse passing the gage on the anvil is approximately 83 per cent of the initial pulse there. The second pulse passing the gage on the specimen does not reach its expected value because fracture of the material occurs at this point before this strain is reached. The strain which exists at this gage when the material fractures there is approximately 65 per cent of the intensity which could have been developed had not fracture occurred. As will be shown later, the stress at this point when the material fractures there far exceeds the static tensile strength of the material.

The oscillogram of Fig. 20 will also serve in demonstrating how the magnitude of the stress in each pulse was determined. In calibrating the gages on the anvil, for a $\Delta R/R$ (the ratio of the change in resistance of the gages to their original resistance) of $1.2\ (10)^{-3}$, the trace on the oscilloscope was deflected 7.0 divisions. Thus, in the oscillogram of

Fig. 20 where the initial pulse through the anvil (the lower trace) caused a trace deflection of 3.9 divisions, the magnitude of stress was

$$S = \left(\frac{\Delta R}{R}\right)\left(\frac{1}{\text{Gage Factor}}\right)(E)\ (\text{No. Divisions})$$

$$= \left(\frac{1.2(10)^{-3}}{7.0}\right)\left(\frac{1}{1.82}\right)(25.8)(10)^{6}\ (3.9) = 9500 \text{ psi}$$

Similarly, in calibrating the gage on the plaster rod of Fig. 20 (upper trace) an introduction of $\Delta R/R$ of $1.6(10)^{-3}$ produced a trace deflection of 7.1 divisions. Since the pulse measured by this gage produced a trace deflection of 5.0 divisions, the magnitude of stress was

$$S = \left(\frac{\Delta R}{R}\right)\left(\frac{1}{\text{Gage Factor}}\right)(E)\ (\text{No. Divisions})$$

$$= \left(\frac{1.6(10)^{-3}}{7.1}\right)\left(\frac{1}{1.82/2}\right)(1.30)\ (10)^{6}\ (5.0) = 1600 \text{ psi}$$

In this particular test (Fig. 20) the pulse traveled down the anvil and returned--a distance of 44 in. in steel--in a time of 4.7 cycles or 235 microseconds. The pulse propagation velocity in the steel was therefore 187,000 in. per sec.

The pulse front arrived at the gage on the plaster in a time of 255 microseconds after passing the gage on the anvil. Calculations using these data show that the propagation velocity in the plaster was 102,000 in. per sec.

A summary of the results obtained from the first set of experiments is shown in Table 2. It will be noted that the transmissibility factor for a pulse passing through an interface between steel and plaster and the reflectability factor for the same barrier correspond favorably to the values determined by the theoretical analysis.

In a second set of investigations it was hoped that by observing the fracture times for the conductive circuits running longitudinally on the rod that information might be obtained whereby crack propagation velocities could be determined, or at least that some qualitative information of the nature of the fracture might be obtained. Photographs of a specimen before and after its fracture are shown in Fig. 21. The upper photograph is of the rod prior to its breakage, and the lower one is of the same rod which was reassembled after having been broken.

It may be observed that in the vicinity of the critical section (the section of the rod first going into tension--located 9.9 in. from the left end in the photograph) that the broken segments are thin wafers and that the segments grow in their longitudinal dimension as the distance that they are removed from that section. Evidence points to the conclusion that fracture first occurred on the broken section nearest the critical section. Within one-half in. of the critical section there existed in every test a broken section of the rod, and it was concluded that this was the first complete fracture.

The type of observation made in this set of experiments is the oscillogram shown in Fig. 22. The top trace is a record of the pulses registered by the gage on the anvil. The second trace from the top registers the breakage of Circuits No. 3 and 4; the upper movement of this trace indicating the breakage of No. 3 and the movement back down indicates the breakage of No. 4. The third trace from the top registers the breakage of Circuits No. 1 and 2 in the same manner. The lower trace

Table 2. Summary of the results of the initial set of experiments

Pulse propagation velocity, steel, in. per sec.	Pulse propagation velocity, plaster, in. per sec.	Transmissibility factor, steel to plaster, per cent	Reflectability factor, steel to plaster, per cent
187,000[a]	100,000[b]	17[c]	-83[c]

[a] This is based on 30 measurements ranging from 182,000 to 192,000 in. per sec.

[b] This is based on 10 measurements ranging from 95,000 to 105,000 in. per sec.

[c] This figure is based on 4 measurements. Deviation was never greater than 5 per cent from the figure.

has recorded two sine waves of 20 kilocycles per sec.; either of these may be used to determine the time that the various events occurred.

Calculations made from data taken from the upper trace show the stress in the initial pulse through the anvil to be 7,950 psi compression. Having established the transmissibility factor for the interface between steel and plaster to be 17 per cent, the transmitted stress in the plaster will then be 1,350 psi compression. Thus the applied stress which will subsequently cause fracture of the specimen is 1,350 psi tension.

The section on which fracture occurred went into tension 526 microseconds after the impact. The sine wave at the bottom of the oscillogram indicates that breakage of the painted strips took place as follows:

Circuit No.	Time after impact, breakage occurred. (microseconds)
3	615
1	625
4	655
2	675

If one assumes that only a single crack existed, that the tip moved across the cross section in a circular front, and that it was propagated at a constant speed, then its origin could be located using Eqs. 42 and 43. The crack propagation velocity of this supposed crack is then found to 10,600 in. per sec. by the use of any of Eqs. 39, 40, or 41.

According to Roberts and Wells (13) this velocity is considerably lower than might be expected; i. e., on the basis of their investigations

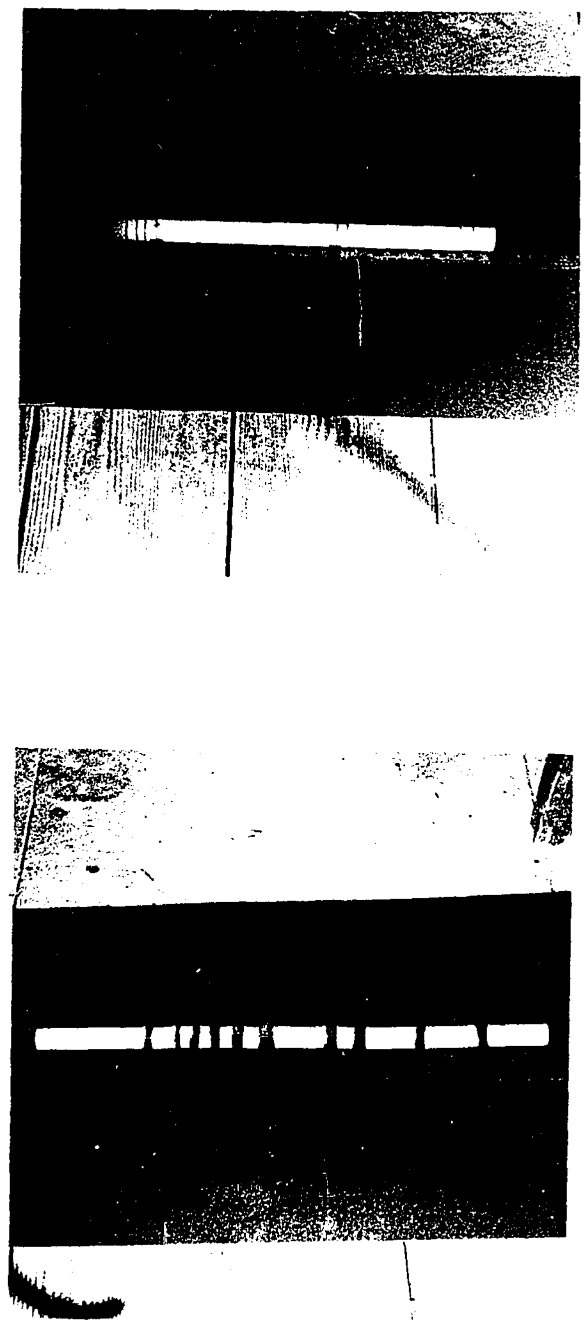


Fig. 21. Photographs of a typical plaster specimen before and after fracture

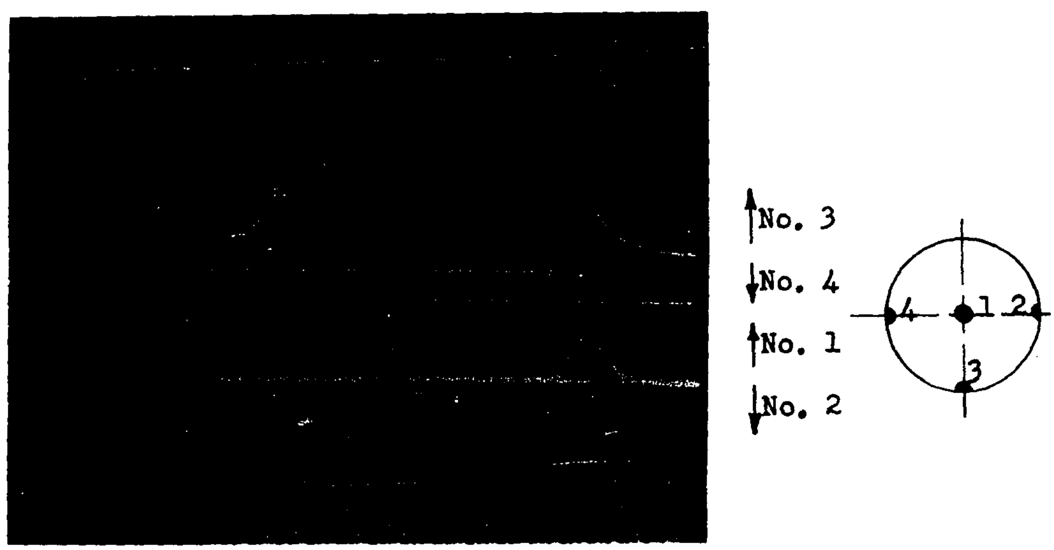


Fig. 22. Oscillogram showing the breakage of the conductive circuits located on the specimen as shown on the drawing on the right

one would expect a value of approximately 38,000 in. per sec. Further investigation of other oscillograms of this type show that this particular oscillogram has yielded a propitious set of data insofar as the determination of crack propagation velocity is concerned. In general, a propagation velocity could not be obtained by this method of loading and observation. Qualitative information as to the nature of the fracture was obtained however.

The determination of the crack propagation velocity was not possible because it appeared that not one but several cracks originated, almost simultaneously, on the cross section and joined together to form the fracture. Either this occurred, or there is the possibility that before fracture was complete on the critical section fracture had commenced on one of the other sections a further distance away.

The fact that several cracks were produced on the cross section in some instances may be inferred from a study of Fig. 23. From the data supplied by this oscillogram it was determined that the tensile stress applied to the specimen was 2,260 psi. The critical section went into tension 526 microseconds after the impact. The breakage of the conductive circuits were taken from the oscillogram and are as follows:

Circuit No.	Time after impact, breakage occurred. (microseconds)
4	580
2	595
3	600
1	615

Since breakage occurred in those circuits located diametrically opposite each other before the center circuit was broken, more than one crack must have existed. This same situation occurred in other tests of this set. In some of the tests which were made using this method of observation, for a single crack to have existed, that crack would have had to originate at some point outside the cross section of the rod--an impossible situation.

Table 3 is a summary of the results of the second set of tests. It will be noted that the tensile stresses applied to the plaster specimens vary from 1,350 to 3,250 psi. This range was dictated by the limitations of the loading machine. The low pressures in the plenum chamber which might have produced lower stresses caused the striker to move spasmodically. The upper stress produced in the plaster corresponded to a stress of 19,000 psi in the steel anvil. A stress of 20,000 psi had been pre-selected as the upper limit for stressing the steel.

Because no clear-cut conclusions could be drawn from the second set of tests as to the effect of intensity of stress on the rapidity of completed fracture, a third set of investigations were instituted. It might seem that the number of cracks initiated might increase with the intensity of the applied stress. If this were the case, because of their close proximity it would take less time for the cracks to join together and for the fracture to be completed.

In order to determine the time of completed fracture the surface of the specimen in the vicinity of the critical section was completely coated with the conductive paint. When this single circuit was broken, it was

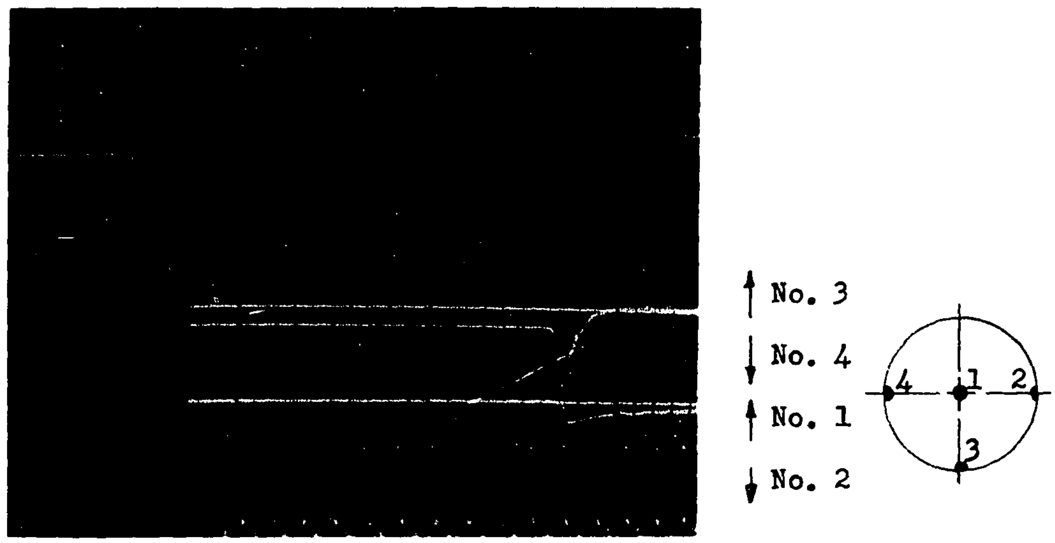


Fig. 23. Oscillogram showing that multiple cracks exist on the cross section of the specimen

assumed that fracture of the rod was complete. Typical of the results obtained in this set of investigations is the oscillogram shown in Fig. 24.

The upper trace in the photograph is the intensity-time plot of the pulses passing the gages on the anvil. The upward movement of the center trace indicates that the circuit has been broken and that fracture is complete. The time occurrence of the events may be ascertained as usual by utilizing either of the two sine waves at the bottom of the photograph.

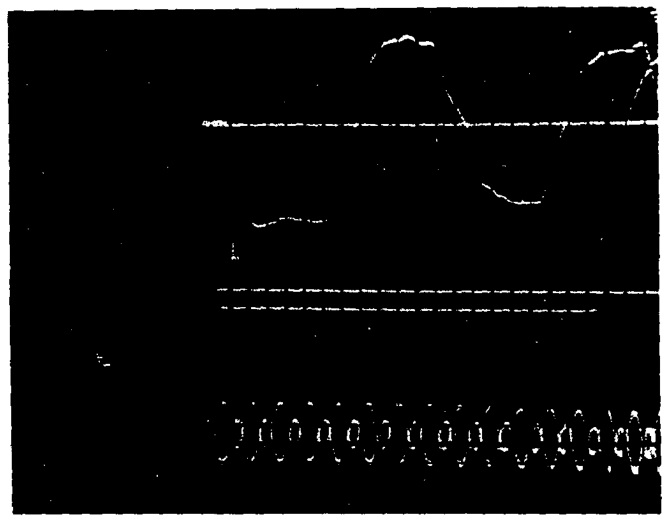
Table 4 is a summary of the results of the last series of tests. The stresses produced during these investigations were within the limitations of the loading machine and were evenly spaced over the allowable range of stress values.

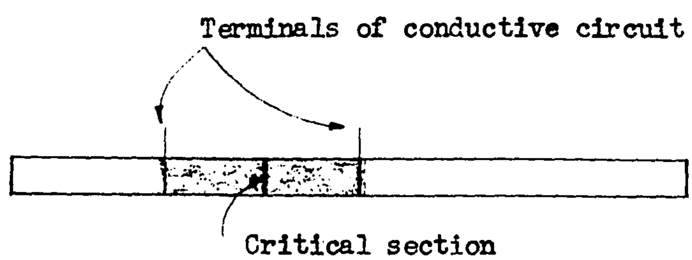
A plot of these results is shown in Fig. 25. Salient features which are identifiable on the figure are:

1. The applied stress which momentarily exists far exceeds the static tensile strength of the material.
2. After the critical section becomes loaded in tension, there is a time interval before completed fracture of the specimen has taken place.
3. For this range of values of applied stress, the time for the fracture to be completed is independent of the applied stress.

Table 3. Summary of the results of the second set of tests

Test No.	Tensile stress, plaster, psi	Time, plaster into tension, microseconds	Time, Circuit No. 1 breaks, microseconds	Time Circuit No. 2 breaks, microseconds	Time Circuit No. 3 breaks, microseconds	Time, Circuit No. 4 breaks, microseconds
T 151	1,350	526	625	675	615	655
T 152	1,640	526	610	570	600	625
T 153	2,150	526	622	560	602	622
T 154	2,260	526	615	595	600	580
T 157	2,550	526	582	560	572	580
T 158	2,720	526	634	624	627	614
T 159	3,250	526	605	580	570	630





Specimen

Fig. 24. Oscillogram showing the instant of completed fracture of the plaster rod

Table 4. Summary of the results of the third set of tests

Test No.	Tensile stress, plaster, psi	Time, plaster into tension, microseconds	Time, completed fracture, micro-seconds
F 4	1,870	526	695
F 1	2,310	526	660
F 3	2,840	526	675
F 6	3,350	526	700

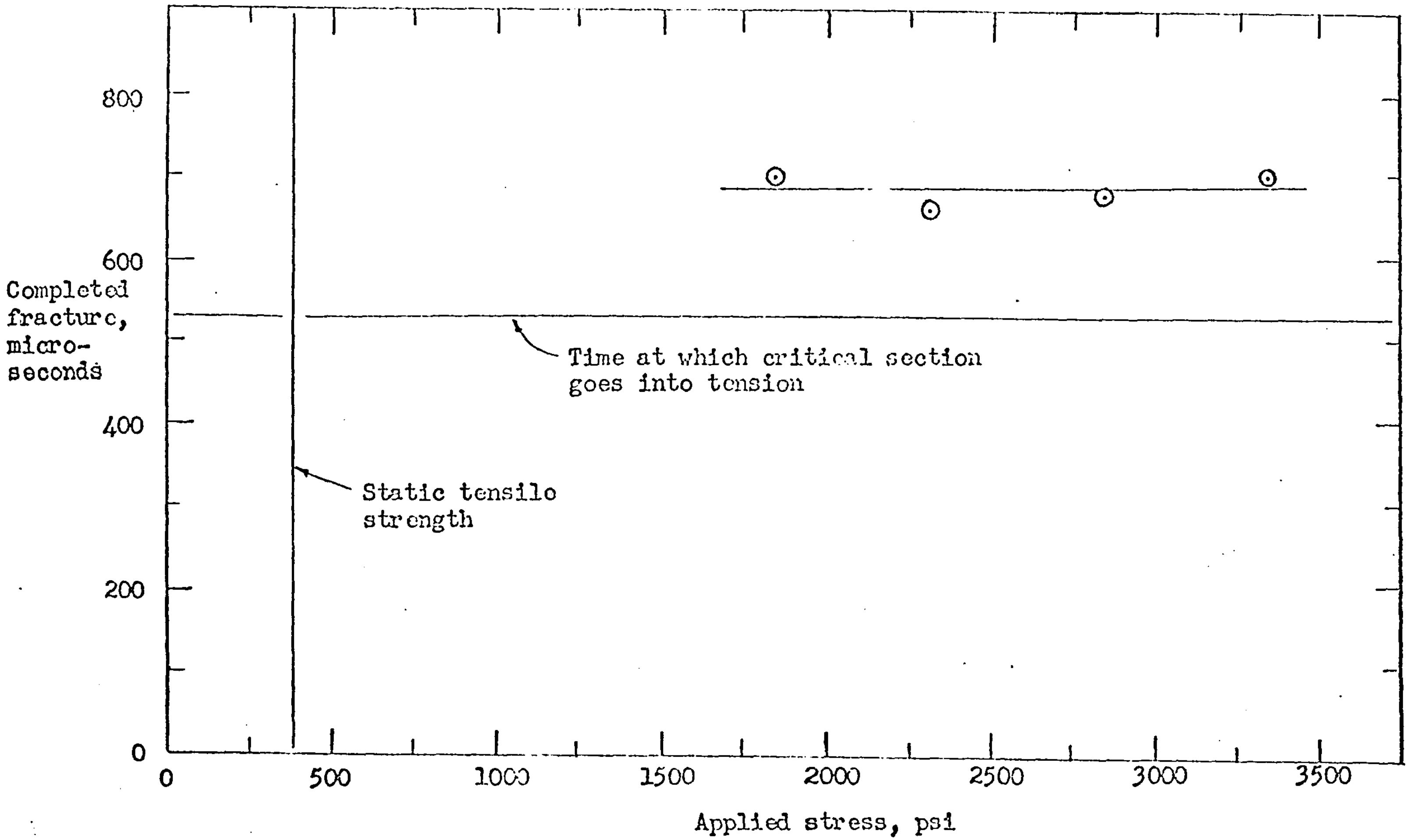


Fig. 25. Time-stress plot of completed fracture of plaster specimen

CONCLUSIONS

The method used in this investigation for producing the impulsive loading in a plaster rod is an effective one. Its effectiveness may be judged by observing an oscillogram of the pulse such as that shown in Fig. 20. Although the time for the pulse to rise to its full value is not instantaneous, it is comparatively short, being on the order of fifteen microseconds. At the section in the plaster rod where fracture occurred, the period of constant intensity of the pulse could have lasted for two hundred microseconds. However, fracture of the section was complete before that amount of time had expired.

The desired loading can be consistently reproduced. A calibration plot of strain intensity in the specimen versus air pressure in the plenum chamber was maintained. Thereupon, the strain intensities which were desired were to be had by controlling the air pressure.

This method of load application produced a pulse behavior in the system which corresponded to the expected behavior as determined by the theoretical analysis. This is conclusively established upon observing the comparison of Figs. 19 and 20. The figures illustrate the comparable state of the assumed mathematical model and the laboratory observations.

Previous investigators had shown the pulse propagation velocity in rods to be equal to the theoretical velocity, and these results confirm their investigations. In the particular case of plaster the dynamic Young's modulus was found to be 1.31 $(10)^6$ psi, and the density was 1.34 $(10)^{-4}$ lb sec^2 in^{-4}. Substitution of these values into the expression for the theoretical velocity, $c = (E/\rho)^{\frac{1}{2}}$, yields a value of 99,000 in. per sec.

All observed values of the pulse propagation velocity in plaster were within two percent of this value.

The theory of the behavior of a pulse moving through a boundary between two materials has been presented herein. No results of other investigators which might have shown the accuracy of such theory have been found. Measurements of the actual strains in the primary pulse and its reflected and transmitted portions have been made on several tests, and the accuracy of the results warrant the acceptance of the theory in the form presented. This theory is as accurate as the theory on the pulse propagation velocity.

The inclusion of multiple fracture circuits on the cross section of the plaster rod allowed an insight to the nature of the fracture process which had not heretofore been known. It is true that photographs of plates of glass and similar transparent materials in the process of being fractured have been made. The photographic technique understandably could not be applied in the fracture of slender plaster rods. The use of multiple fracture circuits of silver paint leads to the conclusion that cracks, as a result of this type of loading, originate at more than one point on the cross section. The observation is made that a finite time interval is required for the several cracks to join together which would complete the severance of two segments of the rod at that cross section.

It would be difficult, if not impossible, to use the multiple fracture circuitry to determine the crack propagation velocities in these experiments. If the object of an investigation is solely to determine the velocity of crack propagation in a material, then a technique similar to that used by Hudson and Greenfield (18) could be successfully employed. By this technique only one crack is produced, and its propagation would not therefore be obscured with the development of additional cracks.

This investigation shows that it is an erroneous belief that fracture occurs when the stress on a section of the material reaches the static fracture strength. It further shows that a material can sustain for a period of time a much higher stress than the static fracture strength. For instance, in every test of this investigation the compressive stress in the plaster was kept below the compressive strength of the material. After the compressive pulse reflected at the free end as a tensile pulse, the tensile pulse had the same magnitude as had the compressive pulse. The stress in the tensile pulse far exceeded, however, the static tensile strength of the plaster. In the case shown in Fig. 20 a tensile stress of 1050 psi may be calculated from the strain registered at the moment fracture occurred at that point. Yet the static fracture strength of the plaster is only 375 psi.

Although the brevity of the duration of loading is one of the characteristics of impulsive loading, these measurements show that time is a significant quantity in the fracture process of a brittle material. It is concluded that crack propagation is initiated whenever the static tensile strength is exceeded. The applied stress may be larger than the static tensile strength, but it would exist at a point only so long as fracture of the material had not proceeded to that point.

The geometry of the broken segments of the specimen in the vicinity of the critical cross section suggests that fracture first occurred on or near that section. The critical cross section is that section upon which a tensile stress sufficient to cause fracture was first applied. If the cross sections contained no imperfections and the material were homogeneous, this section would be the first to be completely fractured.

The results of the third set of experiments of this research show that the time lapse required to complete the fracture is not a function of the magnitude of the stress applied to the rod. It seemed plausible during the early investigations that an increased stress might activate more cracks and that therefore the distance which the individual crack must travel during the fracture process is shorter than when there are fewer cracks. It would follow that the time for completing the fracture would therefore decrease.

It is not evident that more cracks were activated because of the application of the higher stress. The tests that were made fully cover the stress range allowed by the limitations of the impact loading equipment. Within this range the times for complete fracture are independent of the stress magnitude. Evidence of this conclusion is found in Fig. 25.

A summary of the contributions of this investigation are listed below.

1. Measurements were made which validate the theory of pulse transmission and reflection at a boundary between two materials.
2. The method of observing the nature of fracture in slender rods by the use of multiple fracture circuitry was established.
3. A quantitative observation was made which showed that a lapse of time was required to complete the fracture of the rod.

4. A rod can sustain for a short period of time a considerably larger stress than the static fracture strength.
5. Intensity of loading does not hasten the fracture process in the range of stresses applied in this investigation.
6. The character of the fracture process of a brittle material which operates under these conditions of failure has been described.

RECOMMENDATIONS FOR FUTURE WORK

When this research was undertaken, it was hoped, however naively, that some quantitative relationship between the pulse propagation and the crack propagation velocities in plaster could be achieved. This type of work has been done in the last decade by investigators on other materials and using other techniques. In order to accomplish this result some means of loading other than the one used here should be employed. The resulting fracture should be such that it is caused by only one crack, and the propagation of this crack could then be observed so as to determine its velocity. The method used in these experiments for the determination of pulse propagation velocities is a reliable one; it is easily understood and applied, and its use will undoubtedly be continued by investigators in this work.

The symmetry of the assembled broken segments of the rod about the critical cross section lend credence to the reasoning that it was first fractured on or near that section. More conclusive evidence might be supplied if a large number of conductive circuits in the vicinity of the critical section were employed. The electronic apparatus employed in these particular investigations preclude the use of a large number of conductive circuits. One might very well employ the use of high-speed camera techniques to pinpoint that cross section upon which fracture was first completed.

An eventual goal of this type of research might be the development of a better shelter for blast protection. Interpreted in that light one would seek a means of preventing the fracture of the plaster rod in this type of experiment. One such approach might be the search for a means of diminishing the intensity of the pulse transmitted to the plaster from the steel. This might be accomplished by the insertion of a third material between the steel and the plaster which would absorb the pulse or reflect a major portion to the point where that part transmitted would not be harmful.

This problem might be attacked physically from the other end. At the free end of the rod a means might be found for spoiling the reflection of the compressive pulse. The introduction of another material there or the addition of a peculiar geometry might accomplish this goal.

Of the many studies on the propagation of longitudinal strain pulses in long rods it is generally true that they have dealt with rods of circular cross section. By far the majority of the published articles are based on the assumption that plane sections remain plane as the pulse moves through the section. The simplest of structures has been used as a vehicle for the studies of this investigation, and it has been observed that the theory based on this assumption agrees with the results obtained experimentally. It would seem that another direction to take in the study of fracture of plaster rods by longitudinal impact would be the employment of wide rectangular rods as a vehicle for the study. The warping of the cross section with the passage of the pulse might noticeably affect the character of the pulse as it is propagated along the rod, and the nature of the fracture might be noticeably different from that observed for the long circular rod.

Purposely selected for the material of the rods of this investigation was one possessing the linearly elastic property. If a pulse is propagated through a plastic material, because of the variation of Young's modulus each stress increment will propagate at a different velocity. With time the shape of the pulse as it is propagated along the rod will become elongated. This then offers another avenue to the solution of the problem of rendering the pulse harmless. The character of the plastic pulse will become more dispersive in nature than that of a pulse correspondingly produced in the brittle material of these experiments.

SYMBOLS EMPLOYED

A = area of cross section

E = Young's modulus

KE = kinetic energy

L = pulse length

PE = potential energy

R = electrical resistance

S = axial stress

S_r = axial stress of the reflected pulse

S_t = axial stress of the transmitted pulse

T = surface tension

U = stored elastic energy

W = work done against surface tension

a = one-half of crack width

a_o = one-half of critical width of crack

$\dot{a}$ = propagation velocity of a crack

$\dot{a}_T$ = terminal propagation velocity of a crack

c = propagation velocity of a strain wave or of a strain pulse (theoretical velocity)

d = distance between two points on a cross section

k = unspecified constant

p = property of a boundary between two materials

r = radial distance to a point

t = time

u = axial displacement

u_r = lateral displacement

v = particle velocity

θ = angular distance

ϵ = unit strain

ρ = density of material

μ = Poisson's ratio

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APPENDIX

An explanation of the pulse propagations shown in Fig. 18 will be made.

Time measurements commence at the instant of the impact of the striker on the anvil, and those listed on the right of the figure indicate a change of events as noted by the strain gage on the anvil or the strain gage on the specimen.

At impact a pulse, the intensity of which is assumed to be one psi, is propagated to the right along the anvil, and a second pulse of the same intensity is propagated to the left along the striker. Following the course of the pulse propagating to the right, after 69.5 microseconds it has arrived at the strain gage on the anvil. The time interval is calculated as

$$t = \frac{x}{c} = \frac{13}{187{,}000} = 69.5\ (10)^{-6}\ \text{sec.}$$

Similarly, after 328 microseconds this pulse has reached the strain gage on the specimen. This time interval is calculated as

$$t = \frac{35}{187{,}000} + \frac{14.1}{100{,}000} = 328\ (10)^{-6}\ \text{sec.}$$

This time represents a travel of 35 in. through the steel and a travel of 14.1 in. through the plaster. The propagation rate in plaster is 100,000 in. per sec.

The intensity of this pulse has decreased to 0.17 psi compression in the plaster because this is the intensity which is transmitted through the boundary between the steel anvil and the plaster specimen. It will also be noted that at this boundary 0.83 psi tension is reflected and that the front of the reflected pulse moves to the left from the boundary.

After 526 microseconds the pulse first reaching the strain gage on the specimen has reflected as tension from the free end of the specimen and has arrived back at that gage. The total distance of travel for this front has been 35 in. of steel and 33.9 in. of plaster.

The pulse moving to the left along the striker after impact is reflected as tension from the free end of the striker. It then proceeds to the right and is transmitted through the boundary between striker and anvil without a decrease in intensity. The front moves the entire length of the anvil and is transmitted through the boundary between anvil and striker with an intensity of 0.17 psi tension. Thus, at 526 microseconds this front has moved a distance of 37 in. through the striker, a distance of 35 in. through the anvil, and a distance of 14.1 in. through the specimen.

At 526 microseconds the two fronts meet, are superimposed, and the cross section is stressed with a tensile stress of 0.17 psi.